PHY 303, I-Semester 2023/24, Tutorial 4

27. Sept. 2023

Discuss on your table in AIR. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1 Catch up on all items of tutorial 3 that interest you and you did not yet have time to go through.

Stage 2 Discuss the following clarifying exercises depending on which topic you feel you require more clarification:

(i) (ladder operators) Explicitly with all details, the first three Harmonic oscillator eigenstates are

$$\phi_0(x) = \left(\frac{1}{\pi\sigma^2}\right)^{1/4} e^{-x^2/2\sigma^2} \tag{1}$$

$$\phi_1(x) = \frac{\sqrt{2}}{(\pi\sigma^2)^{1/4}} \left(\frac{x}{\sigma}\right) e^{-x^2/2\sigma^2} \tag{2}$$

$$\phi_2(x) = \frac{1}{\sqrt{2}} \frac{1}{(\pi\sigma^2)^{1/4}} \left(\frac{2x^2}{\sigma^2} - 1\right) e^{-x^2/2\sigma^2} \tag{3}$$

Using the ladder operators

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\sigma^2}} \left(x \mp \sigma^2 \frac{\partial}{\partial x}\right), \tag{4}$$

(these slightly differ from those in the lecture/Griffith, but fulfill the same job), show that:

$$\begin{aligned} \hat{a}_+ \phi_n(x) &= \sqrt{n+1}\phi_{n+1}(x), \\ \hat{a}_- \phi_n(x) &= \sqrt{n}\phi_{n-1}(x), \end{aligned} \tag{5}$$

explicitly among those first three states.

(ii) (wavepacket spreading) Consider a free particle treated quantum mechanically, that is at $t = 0$ in a Gaussian wavefunction, Eq. (2.80), at $x_0 = 0$ with spatial width σ. For the following examples, find the time $t_{\text{spr}} > 0$ at which the spatial uncertainty has increased by a factor of two, **using the material of section 2.6.**

(a) An electron with $m_e = 9.1 \times 10^{-31}$ kg and an uncertainty of the size of an atom $\sigma = 5.3 \times 10^{-11}$ m.

(b) A cricket ball of mass $m = 0.15$ kg, with $\sigma = 1$ μm.

(c) An electron with $m_e = 9.1 \times 10^{-31}$ kg and an uncertainty of the size of a nanoparticle $\sigma = 20$ nm.

Stage 3 Bra-Ket Notation: Make the usage instructions for Dirac (Bra-Ket) notation from week 6 accessible on a device.

(a) Then translate the following superposition state wavefunction (assuming the standard particle in the infinite square well potential) into Dirac notation:

$$\Psi(x) = \frac{1}{2}\phi_0(x) + \frac{1}{\sqrt{2}}\phi_1(x) + \frac{1}{2}\phi_2(x). \tag{6}$$

(b) Only using Dirac notation, check whether this state is normalised correctly.

(c) Using Dirac notation, find the probabilities for energy to be E_n, for position to be within $[x, x + dx]$ and for momentum to be within $[p + dp]$. For which cases and how/when do we have to convert back to a more explicit representation of the quantum states at some point, to get an answer? Do that conversion without the final step of evaluating any integrals that pop up.

(d) Write equation (1.71) and other equations required to use it in Dirac notation.