

PHY 303, I-Semester 2023/24, Tutorial 3

6. Sept. 2023

Discuss on your table in AIR on your allocated table number. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

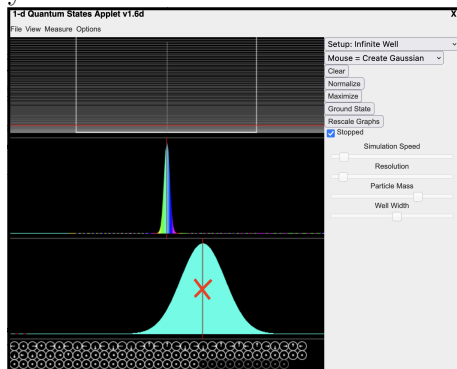
Stage 1 Particle in the box, online app: Start the online-app for exploring solutions of the TDSE at <http://www.falstad.com/qm1d/> on your laptop or on a pad in AIR. Please go through the mini documentation `quantum_app_manual.pdf` provided on teams and/or the movie `quantum_app_manual.mp4` that I made, but ideally also read the more extensive online documentation.

We can now use this app to reproduce example 14 of the lecture. The default setting should be “setup: infinite well”. There should be two white marked circles with rotating arrows at the bottom. These are the $c_{1,2}(t)$ from Eq. (2.19) represented in the complex plane. The height of the curve in the centre panel is $|\Psi_3(x, t)|^2$. The color shading indicates the complex phase $\varphi(x, t)$ of the wave function $\Psi_3(x, t) \equiv \sqrt{n(x, t)}e^{i\varphi(x, t)t}$. It turns out that lots of “color-stripes” indicate higher velocity, uni-color indicates small or no velocity.

- (i) Inspect the eigenstates of the particle in the box, and qualitatively confirm all the results of the lecture, such as the dot-points below Eq. (2.18).
- (ii) Now follow the instructions above to create Example 14 in the app. Discuss what you see. Look at the spatial variation of the phase, and interpret it in terms of a net velocity or probability current (see section 1.6.4.). How does this make sense intuitively?
- (iii) Let us now combine the “particle in the square well potential’ with a Gaussian wavepacket. Adjust all settings as shown in the screenshot below, and click with the mouse near the red-cross, which will create the wavepacket

$$\Psi(x, t = 0) = e^{i\frac{p_0}{\hbar}x} e^{-x^2/(2b^2)} / (\pi b^2)^{1/4}, \quad (1)$$

where p_0 is the momentum at which you have clicked into the panel near the red-cross. Discuss what the wavefunction means. Then discuss what you expect to happen. ONLY then, uncheck “stopped” and confirm your expectation with the simulation. Discuss the physical meaning of what you observe.



- (iv) If the app did not exist, and you had to calculate the time-evolution from the given Gaussian initial state analytically, how would you proceed?

Stage 2 Harmonic oscillator, online app: Now switch the first pulldown menu to “harmonic oscillator”.

- (i) We want to reproduce example 16 (or assignment 2 Q2), on a classical looking oscillation using the app. For that, first create the ground-state of the oscillator. Then switch the second pulldown menu to “mouse, translate function”, and move the ground-state gaussian a bit to the left or right. Then let it play. What do you see? You may also analyze all the panels in terms of our discussion of the Gaussian wavepacket and its Fourier transform in section 2.6.
- (ii) In a slightly different approach, set the second pulldown menu to “mouse, create Gaussian”, and click somewhere on the side from the centre of the oscillator potential. What is different to before? What is the same? Why do the differences happen?

Stage 3 Quantum tunneling: [Stages 3,4 are bonus suggestions, please thoroughly complete 1,2 before doing these]

The app also allows us to have a look at quantum tunnelling, but we have to “embed” the barriers within one of the potentials provided, such as a very large wide well.

- (i) Change setup to “Well pair”. You should see two square wells with a small barrier in between. This barrier is like the one discussed in section 2.2.4., except that here in the app, the space to the right and left is again limited by the far side of the square wells. Then select “Mouse = Create Gaussian” and click on the middle (wavefunction) panel into the right well. Discuss on your table what is happening initially and then after some time (30sec - 1 min). Interpret this with respect to section 2.2.4 lecture material. *Make sure the mass is the default value, else restart the app.*
- (ii) Now use the sliderbar “mass” to increase the mass of the particle and do the above steps again. Increase 2-3 times. Then make it lighter than at the beginning. Discuss what you see. Is this consistent with Eq. (2.40)?
- (iii) Use the slider “well separation” to vary the width of the barrier in the middle. Redo above steps. Discuss what you see and why.

Stage 4 Quantum Reflection: Getting the app to show quantum reflection is a bit tricky, but can be done.

- (i) Change setup to “Coupled well pair”. Click ”stopped” to stop the animation. Then move sliderbar ”well separation” all the way to the right, and sliderbar ”mass” to just on the rhs of the word ”mass”. Switch the second pulldown menu to ”create Gaussian” and click in the bottom most panel

at the same horizontal position as the right edge of the barrier in the top panel.

The bottom panel shows you the Fourier-coefficients (momentum probability density), while the middle one shows you the position space probability density. The red line in the top most panel shows you the mean particle energy. Discuss in your group how you can see in these panels that we created the following initial state for the particle: It has an energy that is larger than the barrier energy $E > U_0$. It is initially most likely at $x = 0$ (in the middle of the screen) but moving to the right.

- (ii) Now unclick "stopped" to start the simulation. If you are impatient slightly increase simulation speed. Discuss what happens once "the particle reaches the potential drop at the edge of the barrier".
- (iii) What would you expect for a classical particle that we start on top of the barrier?