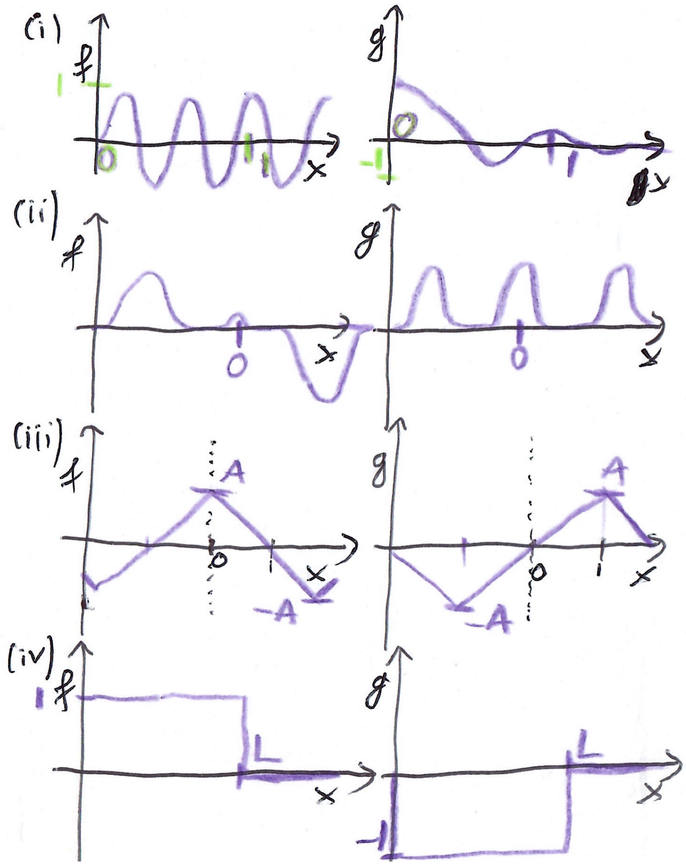
PHY 303, I-Semester 2021/22, Tutorial 2

23. Aug 2023

Discuss on your table in AIR on your allocated table number. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

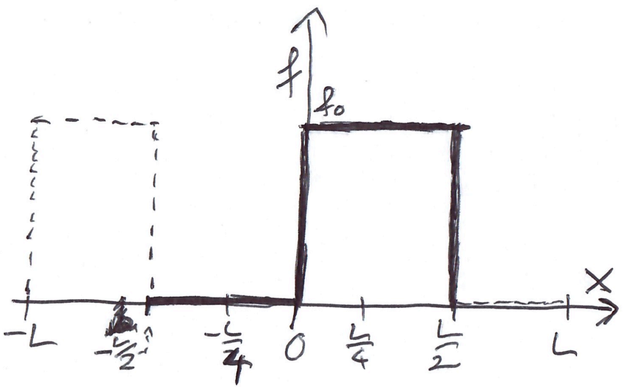
Stage 1 *(functions as vectors)*

(i) Discuss for the function pairs drawn below how orthogonal they are. Rank/order the pairs (i)-(iv) regarding "how orthogonal" they are.



(ii) Expand the function $f(x) = f_0$ for $0 < x < L/2$, $f(x) = 0$ otherwise, in terms of the basis-set $b_n = \{b_n^{(e)}, b_n^{(o)}\} = \{\cos(2\pi nx/L), \sin(2\pi nx/L)\}$ for $n = 0, 1, 2, 3, \cdots$ initially with using drawings. These basis functions are appropriate for periodic functions with period L in between $-L/2$ and $L/2$ [see Eq. (1.20)], so let us assume the function $f(x)$ periodically repeated in this manner as shown by the dashed line. Don't worry about normalisation of the b_n at this point. Draw first the function (as below), then the 5 most slowly varying basis elements, then the first three different terms k in the sequential sum

$$F_k = \sum_{n=0}^{n=k} (f_n^{(e)} b_n^{(e)} + f_n^{(o)} b_n^{(o)}). \tag{1}$$



(iii) Now, let's check our intuition with a calculation of the basis expansion coefficients f_n, in Eq. (1.18). Discuss how to find these coefficients. Now normalisation will turn out to be important, so first normalize the b_n, carefully distinguishing $n = 0$ and $n > 0$. Then find an equation for the coefficients f_n and compare with your guess from (ii).

(iv) Discuss similarities and differences between the above and how you expand a vector $\mathbf{r} \in \mathbb{R}^3$ in terms of a basis.

Stage 2 *(Discretized operators)* Discuss the operator discretisation that we did in example 10 of the lecture for two new operators: (i) the second derivative $\hat{O} = \frac{d^2}{dx^2}$ and the Hamiltonian $\hat{H}$ including some potential $V(x)$. Discuss which matrix you expect. Hint: Either use a finite version of the limit for the second derivative, or matrix multiplication.

Stage 3 *(Normalisation)* Find out which of the following wavefunctions can be normalized and normalise those as suggested in section 1.6.1. You may do any integrations with `mathematica` or Wolfram Alpha . *In either the relevant command is, e.g.* `Integrate[sin[x],x,0,∞]` *for e.g.* $\int_0^\infty dx \ \sin(x)$*, followed by shift-ENTER. You enter* ∞ *as* `ESC inf ESC` *or* `inf` *respectively.*

$$\begin{aligned}
\Psi(r) &= \frac{\cos(r)}{r}, \quad \text{on } 0 \leq r \leq \infty, \\
\Psi(x) &= \theta(D - |x|), \text{ with } D > 0 \text{ on } -\infty \leq x \leq \infty, \\
\Psi(x) &= e^{-|x|} \quad \text{on } -\infty \leq x \leq \infty.
\end{aligned} \tag{2}$$

Above $\theta(x)$ is the Heaviside step function.

Stage 4 Suppose a quantum wavefunction is $\phi(x) \sim f(x)$, where $f(x)$ is the function from Stage 1 (ii), without the periodic repeats at $|x| > L$. Normalise that wavefunction, and then find the expectation value of position and uncertainty of position.