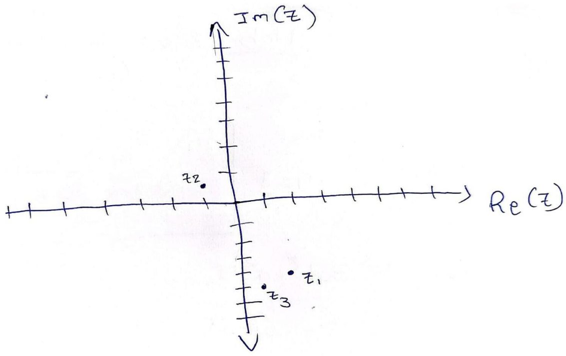
PHY 303, I-Semester 2022/23, Solution Tutorial 1

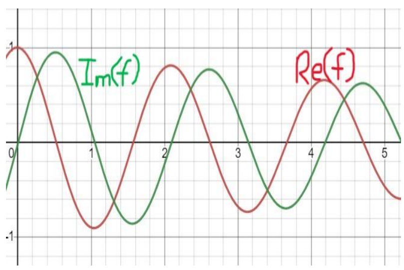
Stage 1 *(math review)* Review your knowledge about complex numbers, vectors and matrices from old course notes, books or the internet. Make sure you would be comfortable to answer the following questions:

(i) Draw the following complex numbers in the "complex plane". Then write them in the form $z = re^{i\varphi}$, for <u>real</u> r and φ:

$$z_1 = 2 - 4i, \qquad z_2 = -1 + \frac{i}{2}, \qquad z_3 = 1 - 5i \tag{1}$$

Make a drawing (sketch) of the function $f(x) = e^{zx}$ for $z = -0.1 + 3i$. *Solution: See z_k below, $f(x) = e^{-0.1x}(\cos(3x) + i\sin(3x)$, using Eq.(1.4).*





$$r_1 = \sqrt{20},\ \varphi_1 = -0.3524\pi,$$
$$r_2 = \sqrt{1.25},\ \varphi_2 = -0.1475\pi,$$
$$r_3 = \sqrt{26},\ \varphi_3 = -0.4371\pi$$

(ii) What is a vector? How do you find the length of a vector? How can you tell if two vectors orthogonal? *Solution: A vector is an element of a vectorspace, which is a set of objects and operations that fulfill axioms in section 1.5.3. We find its length L by taking the scalar product with itself $L = \sqrt{\mathbf{v}\cdot\mathbf{v}}$. Two vectors are orthogonal if their scalar product vanishes $\mathbf{v}\cdot\mathbf{w}$=0.*

(iii) What is a vector space? What is a basis of a vector space? How do you change basis in a vector space? *Solution: See previous point. A basis is* *a minimal set of vectors in terms of which you can write any vector in the vector space as in Eq. (1.10). You change basis using a basis transformation matrix onto each basis vector* $\mathbf{b}' = \underline{\underline{O}}\mathbf{b}$. *These transformation matrices are necessarily orthogonal.*

(iv) How do you add or substract vectors, multiply them with a scalar, or multiply two vectors?

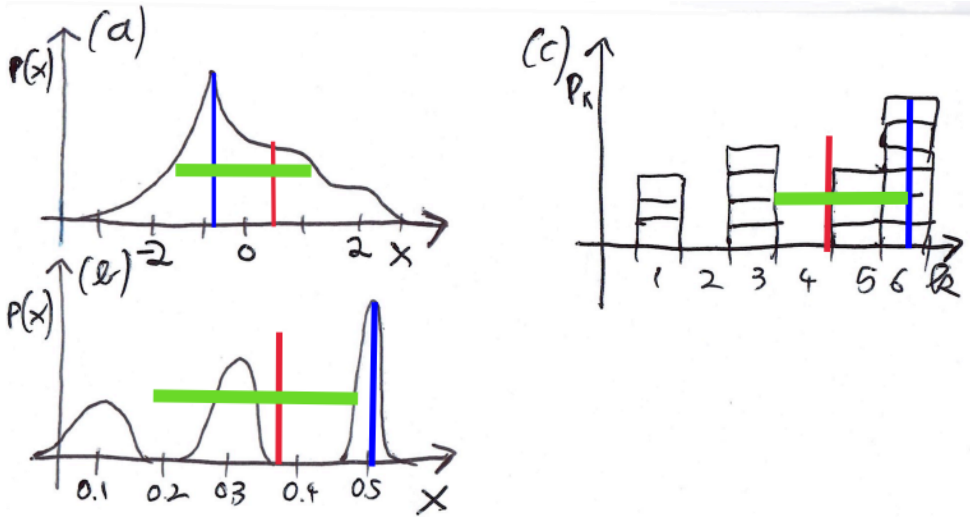
(v) What is a matrix? How do you multiply a matrix with a vector? How do you multiply two matrices? What is the inverse of a matrix?
Solution: A matrix is a representation of a linear map between two vector spaces. You can write it is a square array of numbers. The inverse of a matrix A *is any that fulfills* $A \cdot A^{-1} = \mathrm{I}$ *where* $\mathbb{I}$ *is the unit matrix.*

(vi) Find the eigenvectors and eigenvalues of the following matrix (*try just by "watching". If that does not work, use the usual calculation*). Normalize the eigenvectors:

$$\underline{\underline{M}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \qquad (2)$$

Solution: We see $\mathbf{v}_1 = [0,1,0]^T$ *with* $\lambda_1 = 4$ *directly, by spotting a diagonal entry where the row and column both do not contain any other nonzero entry. Next we see* $\mathbf{v}_2 = \frac{1}{\sqrt{2}}[1,0,1]^T$ *with* $\lambda_2 = 1$ *and* $\mathbf{v}_3 = \frac{1}{\sqrt{2}}[-1,0,1]^T$ *with* $\lambda_2 = -1$*, since there is a little 2x2 block (is more apparent by applying row and column transformations to the matrix to bring the off-diagonal entries closer i.e. the* $(3,1)^{th}$ *and* $(1,3)^{th}$ *element), also with zeros outside, that takes the simple off-diagonal only form like* $\begin{bmatrix} 0 & c \\ c & 0 \end{bmatrix}$.

(vii) In the following drawings of probability distributions, indicate where you would expect the mean value, the most likely value and how large you would expect the standard deviation: *(Solution:) the mean value (red), the most likely value (blue) and how large you would expect the standard deviation (green, length* 2σ*):*



Stage 2 *(physics review)* Review your knowledge about quantum physics PHY106 from old course notes, books or the internet. Make sure you are comfortable with the answer to the following questions:

(i) List at least three key experimental observations that cannot be reconciled with classical physics and thus required the development of quantum mechanics. Discuss why classical physics cannot explain them and (if you know) how quantum mechanics can.
Solution: The photoeffect. The black-body-radiation spectrum (and absence of an ultraviolet divergence), the existence of stable atoms.

(ii) What would you say is the key difference between classical mechanics and quantum mechanics?
Solution: Quantum mechanically, every particle is also wave. Because of that, we cannot attribbute it awell defined position and momentum simultaneously.

(iii) Recall the expressions for (i) Schrödinger's equation, (ii) Heisenberg's uncertainty principle and (iii) the de-Broglie wavelength, then discuss in your team what they mean and what they can be used for.

Stage 3 *(Your expectations)* Within your team, make a list of everything you were told in PHY106 for which you had found the earlier, "low-mathematics-content" explanations unsatisfactory and for which you hope to gain deeper insight in this course. Keep that list and then look at it at the end of the course again.