Phys 637, I-Semester 2022/23, Tutorial 8 9.11.2022

We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Quantum optical master equation)* Revisit the master equation (4.87).

(i) Describe all the *physical* ingredients for the scenario it describes.

(ii) What does each of the terms on the rhs of Eq. (4.89) do/describe? When are they zero/non-zero, large/small?

(iii) When atoms are in $|\,g\,\rangle$ they can change into $|\,e\,\rangle$ either using the Rabi coupling from the Laser or via absorbing a photon from the black-body radiation environment. What is the practical difference in how these processes enter the Master equation (4.89)? What is the physical difference?

Stage 2 *(Steady states)* Derive the results in example 37 for the steady state of an atom under laser drive and spontaneous decay.

Stage 3 *(Adiabatic elimination)* Consider a two level atom as in example 37, Eq. (4.88), but <u>without</u> the environment. We slightly redefine the energies of the states, so that the Hamiltonian is

$$\hat{H}_{\mathcal{S}} = \frac{\Omega}{2}(|\,e\,\rangle\langle\,g\,| + |\,g\,\rangle\langle\,e\,|) - \Delta|\,e\,\rangle\langle\,e\,|. \tag{1}$$

(i) Write down the Schrödinger equation for state amplitudes in $|\,\Psi(t)\,\rangle = c_g(t)|\,g\,\rangle + c_e(t)|\,e\,\rangle$.

(ii) Let us assume that $\Delta \gg \Omega$. Then generalize (in fact it is simpler) the technique of adiabatic elimination to a wavefunction, to get rid of amplitudes c_e and find an equation for c_g only. *Hint: Write down Schrödinger's equation. Assuming $\Delta \gg \Omega$, you set $\dot{c}_e(t) = 0$ (ask me for justification). You can then solve an algebraic equation to find $c_e(t)$ in terms of $c_g(t)$ and insert into equation for $c_g(t)$. What does the equation tell you?*

(iii) Now let us assume the laser intensity is spatially dependent $\Omega = \Omega(\mathbf{x})$. Discuss the meaning of the term you found above. What can we do with it?