Phys 637, I-Semester 2022/23, Tutorial 7 26.10.2022

We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Lindblad Masterequation)* Consider a three-level system with Hamiltonian

$$\hat{H} = E_0|\,0\,\rangle\langle\,0\,| + E_1|\,1\,\rangle\langle\,1\,| + E_2|\,2\,\rangle\langle\,2\,|, \tag{1}$$

and Lindblad operator $\hat{L} = \sqrt{\kappa}|\,2\,\rangle\langle\,2\,|$.

(i) Derive the Lindblad Masterequation for that problem.

(ii) Compare the expected time-evolution for initial states $|\,\phi_a\,\rangle = (|\,0\,\rangle + |\,1\,\rangle)/\sqrt{2}$ and $|\,\phi_b\,\rangle = (|\,0\,\rangle + |\,2\,\rangle)/\sqrt{2}$. How do you interpret this? What parameter(s) govern(s) the time-scale for decoherence? Which don't?

(iii) From the discussion in week6, which type of system environment interaction Hamiltonian is likely responsible for such a Lindblad operator and what would the operator mean physically?

(iv) Derive the Lindblad equation from the Born-Markov equation for the case of Hermitian $\hat{S}_\alpha$ by assuming zero memory time of the environment $\mathcal{C}_{\alpha,\beta}(\tau) = \gamma_{\alpha,\beta}\delta(\tau)$. Use the yellow box on page 76 of the lecture notes for guidance.

Stage 2 *(Quantum Brownian motion)* Let us consider a simpler initial state for quantum Brownian motion than in example 30 of the lecture: $\Psi(x) = \mathcal{N}e^{-\frac{(x-x_0)^2}{2\sigma^2}}$.

(i) Make a sketch of its Wigner function $W(x,p)$ on the board. Based on that discuss how $W(x,p,t)$ should evolve in time for an oscillator without an environment.

(ii) Based on the description of the terms $\sim\gamma$ and $\sim D$ in the Master equations (4.51) and (4.57), how would you expect each to affect that evolution of the Wignerfunction, separately and together? At early times and late times? Use intuition and educated guesses and board drawings, recording your ideas.

Stage 3 *(Perpetually positive density matrices)*

(i) Show that if and only if

$$\langle\,\Psi\,|\hat{\rho}_{\mathcal{S}}(t)|\,\Psi\,\rangle \geq 0, \tag{2}$$

for all possible states $|\,\Psi\,\rangle$, then the populations p_n of the density matrix are ≥ 0 in any basis.

(ii) Show that if the property Eq. (2) is true at time t, then evolution according to Kraus operators as in Eq. (3.66) of the lecture preserves the property at all later times $t' > t$.