Phys 637, I-Semester 2022/23, Tutorial 6 14.10.2022

We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Born-Markov Masterequation)*

(i) What do we try to achieve when deriving a Masterequation? How does it help?
Solution: A masterequation aims to directly propagate the reduced density matrix of the system, without the detour of first finding the complete system+environment evolution operator $\hat{U}(t)$. Since the system is typically much more tractable (e.g. in terms of Hilbert-space basis), this is simpler once it is achieved.

(ii) Discuss in your own words what is the content of the Born and Markov approximations. Explain this once in terms of physics and then what is the consequence in terms of mathematics.
Solution: (Born physics) In the Born approximation we assume the interaction between system and environment is weak and the environment very large. The system thus can only have a minor effect on the environment. (Born mathematics) This allows us to replace the time evolving combined density matrix $\hat{\rho}(t) \approx \hat{\rho}_{\mathcal{S}}(t) \otimes \hat{\rho}_{\mathcal{E}}(0)$, which is a twofold massive simplification: The environment no longer time evolves, and the state remains (approximately) separable. (Markov physics) We assume environment correlations $C(\tau)$ are only nonzero for short delays τ, which are much shorter that typical timescales on which the system density matrix $\hat{\rho}_{\mathcal{S}}(t)$ evolves (=changes). This can be interpreted as the memory quickly forgetting any small effect the environment had on it earlier. (Markov math) Firstly we can set $\hat{\rho}_{\mathcal{S}}(t') \approx \hat{\rho}_{\mathcal{S}}(t)$ in the integration over the delays, since $\hat{\rho}_{\mathcal{S}}(t)$ will be approximately constant (it evolves on much longer timescales than when $C(\tau)$ drops to zero). For a similar reason we can extend the integration domain of delays from t to ∞, since at large delays, $C(\tau) = 0$ anyway so we are not changing the physics but making the math easier.

(iii) Through which quantity does the environment enter the effective system evolution in the master-equation? Which information from the environment enters this quantity?
Solution: All information about the environment that enters the evolution of the reduced density matrix of the system is encapsulated in the environment self correlation function(s). The correlation functions however depend on almost all information available on the environment: The environmental part of the coupling Hamiltonian $\hat{E}$, eigenfrequencies of the environment (due to the interaction picture evolution $\hat{E}^{(I)}$, and the initial

(permanent) state of the environment.)

Stage 2 *(Lindblad Master equation)*

(i) What is good about a Lindblad masterequation and not so good about its precursor, the Born-Markov masterequation?
Solution: The Lindblad masterequation guarantees that the time-evolving reduced system density matrix remains completely positive, such that all populations $\rho_{aa} \geq 0$ for any state $|\,a\,\rangle$. This is NOT guaranteed for the Born-Markov masterequation, as an artefact of the approximations we made to reach it.

(ii) What is meant by a CPTP map?
Solution: A CPTP map is a matp $\mathcal{L}$ that is "completely positive and trace preserving". Hence we we evolve the density matrix with it $\hat{\rho}(t+dt) = \hat{\rho}(t) + dt\mathcal{L}[\hat{\rho}(t)]$, if the density matrix at time t was completely positive (see (i)) and properly normalized $Tr[\hat{\rho}(t)] = 1$, both will be true at time $t + dt$ as well. Note that the Masterequation (whether Lindblad or Born-Markov) is <u>linear</u> in $\hat{\rho}$, which means that you manually have to ensure that you start with $Tr[\hat{\rho}(t)] = 1$, because any other messed up normalisation will also be preserved.

(iii) Describe the ingredients of a Lindblad equation and methods for its solution.
Solution: We need the system Hamiltonian, possibly with a Lambshift $\hat{H}'$ and all Lindblad operators of the problem $\hat{L}_\mu$. The latter are typically defined such that they directly contain the coefficients $\kappa_m u$ in (4.25) of the lecture. Otherwise we would need the κ_μ as well. After writing down the Lindblad equation explicitly in terms of matrix elements of the density matrix $\dot{\rho}_{nm} = \cdots$, it turns in a system of coupled differential equations of first order in time, and can be solved with standard methods that you know for these. I.e. writing it as a matrix DE an using matrix exponentials, or decoupling via tricky addition or subtraction of component equations, or series expansions. Most frequently though, you would just numerically solve them on a computer.

Stage 3 *(Dephasing)* Take a good look at example 27 (Lindblad equation for dephasing) of the lecture. *Solution: Please do and ask questions if unclear.*