Phys 637, I-Semester 2022/23, Tutorial 5 16.9.2022 (mid-sem prep tutorial)

Ignore the ordering or numbering below. Pick questions where you are not so sure and skip the ones where you are sure you know the answer.

Topic 1 *(Basics)*

(i) What is coherence? Which types of coherence are there? What is decoherence?

(ii) Why do we introduce density matrices? What are pure versus mixed density matrices? What are reduced density matrices? Convert the pure state for the harmonic oscillator $|\,\Psi\,\rangle = (|\,4\,\rangle + |\,6\,\rangle)/\sqrt{2}$ into a density matrix. What is the normalisation condition of a density matrix? What is the meaning/interpretation of diagonal versus off-diagonal elements of a density matrix?

(iii) What is a thermal density matrix? What is its interpretation? Which type of density matrix is it?

Topic 2 *(Decoherence)*

(i) Suppose we can solve the complete quantum many body evolution for a system + environment model. How do we look at coherence features or decoherence dynamics in the system?

(ii) Consider a bi-partite system-environment state? How does the coherence between system states depend on the environmental part?

Topic 3 *(Quantum measurements)*

(i) What is meant by quantum or von-Neumann measurement? Contrast the description of a measurement using the von-Neumann scheme or the usual scheme. Why does one introduce the former?

(ii) What is meant by the "measurement problems" and how are they resolved or not resolved?

(iii) What are pointer states?

Topic 4 *(Practicing techniques)*

(i) Revisit any part of quiz1 that troubled you, look at the solution, and then do it as quickly as possible. Ask us questions about all unclear pieces.

(ii) Given a density matrix, how do we find out if it is pure or mixed? Do this for these examples:

$$\rho_1 = \frac{1}{4}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 2 & \sqrt{2} \\ 1 & \sqrt{2} & 1 \end{pmatrix}, \qquad \rho_2 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{1}$$

(iii) For the following explicit and formal density matrices of systems A and B, find the reduced density matrix of A.

- Two spin 1/2, with basis ordering $\{|\uparrow,\uparrow\rangle, |\uparrow,\downarrow\rangle, |\downarrow,\uparrow\rangle, |\downarrow,\downarrow\rangle,\}$

$$\rho = \frac{1}{16}\begin{pmatrix} 3 & -\sqrt{3} & 3\sqrt{3} & -3 \\ -\sqrt{3} & 1 & -3 & \sqrt{3} \\ 3\sqrt{3} & -3 & 9 & -3\sqrt{3} \\ -3 & \sqrt{3} & -3\sqrt{3} & 3 \end{pmatrix}. \tag{2}$$

- A: spin 1 particle and B: harmonic oscillator, with basis $|\, m, n\,\rangle$, and $m \in \{-1, 0, 1\}$, $n \in \mathbb{N}_0$.

$$\rho = \sum_{m,m';n,n'} c_m c^*_{m'} e^{-\frac{|\alpha_m|^2+|\alpha'_m|^2}{2}} \frac{\alpha_m^n \alpha_{m'}^{*n'}}{\sqrt{n!}\sqrt{n'!}} |\, m, n\,\rangle\langle\, m', n'\,| \tag{3}$$

with $\sum_m |c_m|^2 = 1$ and $\alpha_m \neq \alpha_{m'}$ for $m \neq m'$.

(iv) Find the purity in the first case. What does this tell you about entanglement between A and B? In the second case just discuss simple cases that make the density matrix pure or mixed.

(v) For the following superposition state of the harmonic oscillator, write the explicit position space density matrix as a 2D function: $|\,\Psi\,\rangle = (|\,0\,\rangle + |\,2\,\rangle)/\sqrt{2}$

(vi) For the following mixture, write the explicit position space density matrix as a 2D function: $|\,\Psi\,\rangle = (|\,0\,\rangle\langle\,0\,| + |\,2\,\rangle\langle\,2\,|)/2$

(vii) Construct the thermal density matrix for some states $|\,a\,\rangle$, $|\,b\,\rangle$ and $|\,c\,\rangle$, with energies $E_a = 0$, $E_b = E$ and $E_c = 2E$ with some constant E at temperature $k_B T = E$ explicitly as a normalised 3 by 3 matrix.