Phys 637, I-Semester 2022/23, Tutorial 4 14.9.2022

We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Pointer states)*

(i) Consider the following System-Apparatus-Environment Hamiltonian, for a system spin, apparatus harmonic oscillator and multiple environment oscillators

$$\mathcal{H}_{\mathcal{S}} = \Delta E \hat{\sigma}_z, \qquad \mathcal{H}_{\mathcal{A}} = \hbar\omega \hat{a}^\dagger \hat{a}, \qquad \mathcal{H}_{\mathcal{E}} = \sum_n \hbar\omega_n \hat{b}_n^\dagger \hat{b}_n,$$
$$\mathcal{H}_{\mathcal{SA}} = \kappa_{SA} \hat{\sigma}_y (\hat{a} + \hat{a}^\dagger), \qquad \mathcal{H}_{\mathcal{AE}} = \sum_n \eta_n (\hat{a} + \hat{a}^\dagger)(\hat{b}_n^\dagger + \hat{b}_n). \tag{1}$$

Find the pointer states (of the system, with respect to the apparatus), and the pointer states (of the apparatus, with respect to the environment).

Stage 2 *(Wigner function)*

(i) Revise the properties and purpose of the Wigner function that we stated in the lecture.

(ii) Based on that, <u>without</u> a calculation, discuss qualitatively how you think the Wigner function of the following quantum states should look like, and make a drawing (isocontours) of them in the $x - p$ phase-space:

- The $n = 5$ eigenstate of the 1D quantum harmonic oscillator $\phi_5(x)$.
- A plane wave e^{ikx}.
- An Airy function $Ai(x)$.

(iii) (later at home) Corroborate your thoughts from the tutorial with actual calculations, or numerical plots.

Stage 3 *(Schmidt-decomposition)*

(i) Using the Schmidt-decomposition (3.48), obtain expressions for the reduced density matrices for system $\mathcal{A}$ and system $\mathcal{B}$. Show that these reduced density matrices are Hermitian and find their eigenvalues and eigenvectors. With that, show that the Schmidt-decomposition for any state $|\,\Psi\,\rangle$ involving systems $\mathcal{AB}$ can be found by finding the two reduced density matrices in any basis and diagonalising them.

(ii) Does the two qubit state $|\,\Psi\,\rangle = (|\,00\,\rangle + |\,01\,\rangle + |\,10\,\rangle + |\,11\,\rangle)/2$ take the form of Schmidt decomposition? Why/why not? If not, which is a Schmidt decomposition?

(iii) Now consider, as an example for (i), two spin-1/2 particles, labelled A and B, in a (normalized) pure state,

$$| \Psi \rangle = (| \uparrow\uparrow \rangle + | \downarrow\uparrow \rangle + | \uparrow\downarrow \rangle - | \downarrow\downarrow \rangle)/2 \tag{2}$$

Obtain the Schmidt-decomposition by computing and diagonalizing the two reduced density matrices.