Phys 637, I-Semester 2022/23, Tutorial 3 2.9.2022

We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Leftover tutorial 2)* Presumable many of you might not have done stages 2 and 3 of the tutorial 2 sheet, if so, please do them now first.

Stage 2 *(Classical and quantum uncertainties)* Revisit the mixture of coherent states discussed in Q2 of tutorial 2 (please reread the physical discussion of its origin on the tutorial sheet), where

$$\hat{\rho} = \int_0^\infty dr\ p(r) |\, \alpha(r) \,\rangle \langle\, \alpha(r) \,|, \tag{1}$$

with $p(r) = \frac{1}{\sqrt{2\pi}\Delta r} e^{-\frac{(r-r_0)^2}{2\Delta r^2}}$.

(i) Find the position uncertainty $\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$ in that density matrix. Discuss the meaning of the formula that you obtain.
Hints: For evaluating quantum expectation values, use the position representation of the coherent state $\langle\, x \,|\, \alpha \,\rangle = \mathcal{N} e^{-\frac{(x-r)^2}{2\sigma^2}}$ *and known facts about the mean* ($r \sim \alpha$) *and variance* (σ^2) *of this Gaussian wavefunction.*

(ii) (extension outside tutorial) With a computer program, you can plot all density matrix elements ρ_{nm} of $\hat{\rho} = \sum_{nm} \rho_{nm} |\, n \,\rangle \langle\, m \,|$ from Eq. 1 as for example colorful squares in an $N \times N$ array (use e.g. `pcolor.m` matlab). Do that for parameters $\hbar = m = \omega = 1$ (hence $\sigma = 1$) and $r_0 = 10\sigma$, $\Delta r = 4\sigma$ and discuss. You may first have to identify a suitable N to make sure all coherent states are represented correctly.