Phys 637, I-Semester 2022/23, Tutorial 2 24.8.2022

Stage 1 *(Density matrices)*

(i) Motivation for the density matrix concept: Consider two spins, where the first is the "system", and try to devise a "reduced <u>state</u> of the system only", that has the property that it correctly describes all expectation values of operators on the system. Let us take:

$$\begin{aligned} |\,\Psi\,\rangle &= \sqrt{\frac{2}{6}}|\uparrow,\uparrow\rangle + \frac{1}{\sqrt{6}}|\uparrow,\downarrow\rangle + \frac{1}{\sqrt{6}}|\downarrow,\uparrow\rangle + \sqrt{\frac{2}{6}}|\downarrow,\downarrow\rangle, \\ \hat{O}_1 &= \sigma_x^{(1)}, \end{aligned} \tag{1}$$

as a test case *Hint: This should not work in general. In case you find a construction where it does for this example, try it on another one.*

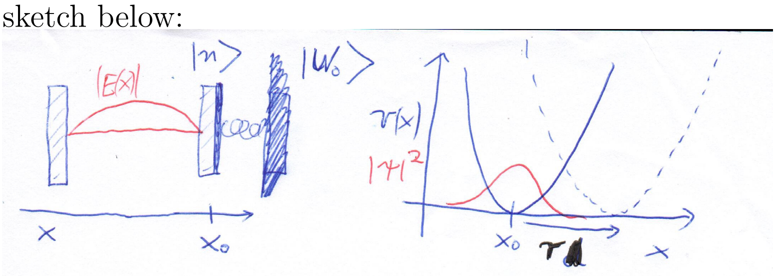
(ii) For the following density matrices of a two-spin system, calculate the reduced density matrix for the first spin only, and then its purity and von-Neumann entropy. We used the basis ordering: $\{|\uparrow,\uparrow\rangle, |\uparrow,\downarrow\rangle, |\downarrow,\uparrow\rangle, |\downarrow,\downarrow\rangle\}$. Discuss.

$$\hat{\rho}_1 = \frac{1}{4}\begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}, \qquad \hat{\rho}_2 = \frac{1}{4}\begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix}. \tag{2}$$

Also write the state from (i) as a density matrix.

(iii) Pick one of the density matrices in (ii), and change basis to left/right for each spin.

Stage 2 Consider an opto-mechanical quantum harmonic oscillator as shown in the sketch below:



Suppose an experiment succeeded to cool that oscillator to the quantum ground-state and then suddenly (approximately instantaneously) displace the minimum of the potential by the distance r as shown. This could be done e.g. by generating a constant radiation pressure force due to light in an optical cavity. This sudden shift creates a coherent state with parameter $\alpha = r/(\sqrt{2}\sigma)$, where $\sigma = \sqrt{\hbar/(m\omega)}$ is the zero point width. First write the density matrix of the oscillator in that coherent state.

Now suppose due to technical noise, the displacement parameter r in the experiment is itself normally distributed with variance $(\Delta r)^2$ and mean r_0. Propose a density matrix in terms of oscillator eigenstates $|\, n\,\rangle$ that describes the experimental situation after the shift, including this imperfection.

Stage 3 *(Von-Neumann measurements)* Consider a simple quantum optics experiment, where you prepared a very short light pulse from a laser beam (that would be in a coherent state wrt. photon number n) and then measure the number of photons with a detector that has single photon resolution. Describe the measurement in terms of the Von-Neumann scheme using steps $\rightarrow$. Define appropriate detector states to write this schematic. Also include the experimenter's brain in the scheme. How does this scheme differ from measurement postulate P3? Which property of Schrödinger's equation is crucial?

Stage 4 *(Bonus)* For the density matrix in Stage 2 including technical imperfections, find a formula for the position uncertainty.