Phys 637, I-Semester 2022/23, Tutorial 1 10.8.2022

Work in the same teams as for assignments. We suggest to do "Stages" in the order below, feel free to change that as per your interests. Discuss first on your table within your team, then with neighboring tables.

Stage 1 *(Course motivation)*

(i) What is an open quantum system? Make a list of some quantum systems you can think of, and for each of them decide whether you have to treat it like an open quantum system or whether you can approximate it as a closed system. Before starting that, think whether besides the type of system, some other information might be important for this classification. Which questions on "open quantum systems" do you have that you want this lecture to answer? *Of course you will be able to answer all these questions really only after the lecture. The idea here is to brainstorm your gut instincts, and later compare.*

(ii) What is meant by coherence? Make a list of coherent wave phenomena versus incoherent ones. Which types of coherence do you know. What open questions on coherence do you have?

Stage 2 *(Quantum mechanics)*

(i) Describe the essential formal basis of quantum physics, and what the major differences to classical physics are. List some successes of quantum mechanics. What shortcomings of quantum mechanics do you know?

(ii) How do we go from single-particle quantum mechanics to many-particle quantum mechanics? Why does it typically get nasty?

(iii) Write the following quantum states (some many-body) as an equation:

- One electron (3D position $\mathbf{r}_1$) is near a proton in the ground-state of the Hydrogen atom, and a second electron is free and has a well defined momentum $\mathbf{p}_2$.
- An electron moving in one dimension travels to the left (momentum $-p$) if its spin projection along the x-axis is $m_s = +1/2$ and to the right (momentum $+p$) otherwise. Travelling to the right is twice as likely as to the left.
- One harmonic oscillator is in an arbitrary state, and a second harmonic oscillator is in the exact same quantum state.
- One harmonic oscillator is in an arbitrary state, but if you measure the energy of a second harmonic oscillator it always is found to be the same as the energy of the first.
- Which of the above states are entangled?

(iv) Write the following Hamiltonian of two interacting particles (different masses m and M) in 1D in terms of their ladder operators (defining these explicitly in terms of quantities given).

$$\hat{H} = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial r_2^2} + \frac{1}{2}M\omega_1^2 r_1^2 + \kappa r_2^2 + \eta(r_1 - r_2)^2. \qquad (1)$$

(v) Interpret the following pieces of a many-body Hamiltonian (interactions) as a sentence that describes the physics of the interaction in question (Pauli matrices refer to a spin 1/2 object, ladder operators to another object or the motional degree of freedom of the same object, treated as a harmonic oscillator).

- $H_{\text{int}} = \hat{\sigma}_z(\hat{a} + \hat{a}^\dagger)$.
- $H_{\text{int}} = 2\hbar\omega|\uparrow\rangle\langle\uparrow|\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega|\downarrow\rangle\langle\downarrow|\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$.
- $H_{\text{int}} = \kappa\left(|\uparrow\rangle\langle\downarrow|\hat{a} + |\downarrow\rangle\langle\uparrow|\hat{a}^\dagger\right)$.

Stage 3 *(Markovianity)* Come up with some further examples of Markovian and non-Markovian stochastic processes, beyond the two given as example in the lecture notes.