Phys 637, I-Semester 2022/23, Assignment 5

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Due-date: 28.10.2022

(1) Let's build us a quantum computer (theorist version): The register of a quantum computer with 2 qubits can be described by the basis $|\,00\,\rangle$, $|\,01\,\rangle$, $|\,10\,\rangle$, $|\,11\,\rangle$. Let's call the first qubit A and the second B, i.e. $|\,00\,\rangle$ really is $|\,0_A 0_B\,\rangle$. It has been shown that, for a quantum computer to do any arbitrary computation, is sufficient if it can perform arbitrary single qubit operation[1] and one of a set of universal two qubit gates [see wikipedia] . One of the latter is the controlled-Z gate, with the truth-table:

$$\begin{aligned} |\,00\,\rangle &\rightarrow |\,00\,\rangle, \\ |\,01\,\rangle &\rightarrow |\,01\,\rangle, \\ |\,10\,\rangle &\rightarrow |\,10\,\rangle, \\ |\,11\,\rangle &\rightarrow -|\,11\,\rangle. \end{aligned} \tag{1}$$

Let our quantum computer have the Hamiltonian

$$\hat{H} = \sum_{n\in\{A,B\}} \frac{\Omega^{(n)}(t)}{2}\hat{\sigma}_x^{(n)} - \frac{\Delta^{(n)}(t)}{2}\hat{\sigma}_z^{(n)} + U(t)(\hat{\sigma}_z^{(A)} - \mathbb{1})(\hat{\sigma}_z^{(B)} - \mathbb{1}), \tag{2}$$

where we interpret each qubit as a pseudospin $\{|\,0\,\rangle \leftrightarrow |\downarrow\rangle, |\,1\,\rangle \leftrightarrow |\uparrow\rangle\}$ so that we can use a Pauli matrix notation, and $^{(n)}$ implies that an operator acts on qubit $n \in \{A, B\}$ only. The control parameters $\Omega^{(n)}(t)$, $\Delta^{(n)}(t)$ and $U(t)$ are written explicitly time dependent, so that a sequence of different operations can be performed.

A real quantum computer will also suffer from decoherence due to the external environment. Let that be described by Lindblad operators $\hat{L}_{\mu=1} = \sqrt{\gamma_A}\hat{\sigma}_z^{(A)}$ and $\hat{L}_{\mu=2} = \sqrt{\gamma_B}\hat{\sigma}_z^{(B)}$, where the γ_n are decoherence rates.

(1a) Ignoring decoherence, design a sequence for control parameters (that means a specific form for the time dependent functions $\Omega^{(n)}(t)$, $\Delta^{(n)}(t)$ and $U(t)$) that implements the following:

(i) Starting in the initialstate $|\,\Psi_{ini}\,\rangle = |\,00\,\rangle$, initialise qubit A in the register separately into a non-trivial (exemplary picked) qubit state. Hence the final state for this part should be $|\,\Psi_1\,\rangle = (c_{A0}|\,0\,\rangle + c_{A1}|\,1\,\rangle) \otimes |\,0\,\rangle$, with $|c_{A0}|^2 + |c_{A1}|^2 = 1$.

(ii) In the next step, initialise qubit B into another non-trivial qubit state. Hence the final state for this part should be $|\,\Psi_2\,\rangle = (c_{A0}|\,0\,\rangle + c_{A1}|\,1\,\rangle) \otimes (c_{B0}|\,0\,\rangle + c_{B1}|\,1\,\rangle)$ with $|c_{B0}|^2 + |c_{B1}|^2 = 1$.

[1] That means it can map any input state $|\,\Psi_A\,\rangle = c_0|\,0_A\,\rangle + c_1|\,1_A\,\rangle$ with $|c_0|^2 + |c_1|^2 = 1$ into any other state $|\,\Psi'_A\,\rangle = c'_0|\,0_A\,\rangle + c'_1|\,1_A\,\rangle$, similarly for B.

(iii) On this register state, subsequently operate with the controlled-Z gate above.

Hints: Global phases do not matter. Revisit Rabi oscillations in two level systems. Maybe read up on the concept of a "Bloch sphere". For finite duration $\tau < T$ pulses of some Rabi frequency such that $\int_0^T dt'\Omega(t') = \pi/2,\ \pi,\ 2\pi$ we call this a $(\pi/2,\ \pi,\ 2\pi)$-pulse. One knows that starting from e.g. state $|\,0\,\rangle$ and $\Delta = 0$, the outcomes of these pulses are $(|\,0\,\rangle - i|\,1\,\rangle)/\sqrt{2}$, $-i|\,1\,\rangle$ and $-|\,0\,\rangle$ respectively. Also consider separately which effect the Ω and the Δ terms have on a wavefuction .[5 points].

(1b) Work out the Lindblad Master equation (4.25) for this quantum computer. Describe in words what the terms $\sim \gamma_{A/B}$ are doing and what you expect their physical effect to be. [5 points].

(2) Bath correlation functions Consider an environment of Nnon-interacting harmonic oscillators and numerically explore their bath-correlation functions.

(2a) Make a mathematica script that generates masses, frequencies and system-oscillator coupling constants for N oscillators randomly from some specifically chosen distribution. For example generate many random frequencies ω_j, such that the probability for $\omega < \omega_j < \omega + \Delta\omega$ is given by the probability distribution function $P(\omega)\Delta\omega$ (for some choice of $P(\omega)$). Plot histograms of those random numbers to check that your distribution is implemented correctly. Then assemble the resultant correlation function $C(\tau)$ for zero temperature and some meaningful nonzero temperature T. [6 pts]

(2b) Explore some different distribution shapes, means and widths and varying N. Particular find out how the width (standard deviation) of the frequency distribution $\Delta\omega$, the number of oscillators N and the temperature T affect the correlation function [4 pts].

Hints: Let us assume we have chosen some dimensionless units so that all parameters are of order "one". This still means we can let them vary between e.g. 0.01 *and* 100 *or such. Then, it might be ok to set all masses equal to one (why is that?). Check the documentation of the mathematica commands* ***Table****,* ***RandomVariate****,* ***NormalDistribution****,* ***MixtureDistribution****,* ***ChiSquareDistribution****,* ***Histogram****,* ***PDF****,* ***Sum****,* ***ListPlot****. Initially start with $N = 20$, but in the end try to crank it up to as high as $N = 2000$.*

(3) Simulating decoherence in a quantum computer The code `Assignment5_program_draft_v1.xmds` is set up to numerically solve a Lindblad equation, for the density matrix of question 1:

$$\hat{\rho}(t) = \sum_{ab;cd\in\{0,1\}} \rho_{ab;cd}(t)|\,ab\,\rangle\langle\,cd\,|. \tag{3}$$

(3a) Implement your Lindblad equation from Q1b at XXX and your time sequence for initialisation of the initial state and controlled-Z gate at ZZZ. Some manipulations on A and B could be done in parlallel, best do them sequentially. Also derive the energy as a function of $\hat{\rho}$ and implement that at YYYY. [4 points].

(3b) Check that your density matrix remains normalised properly and that energy is conserved for constant parameters and no dephasing only, using `Assignment5_plot_checks_v1.m`. Discuss what happens to these checks if parameters are varying in time, or you add dephasing, and why. *Hint: In the likely case that none of that works on first attempt, use `Assignment5_plot_populations_v1.m` to check what your control parameters and populations are doing, `Assignment5_check_hermiticity_v1.m` to make sure the density matrix remains Hermitian, in case it doesn't, use `Assignment5_check_coherences_v1.m` to find the culprit.* [2 points].

(3c) Check that your sequence correctly does steps (i) and (iii) from Q1a, using `Assignment5_check_quantum_computer_v2.m`. , if there is no decoherence ($\gamma_n = 0$) [2 points].

(3d) Check how this changes when decoherence is added. How large can γ_n be before your quantum computer breaks down. What does that mean for the design principles of quantum computers? [2 points].