Week 4

PHY 435 / 635 Decoherence and Open Quantum Systems
Instructor: Sebastian Wüster, IISER Bhopal, 2018

These notes are provided for the students of the class above only. There is no warranty for correctness, please contact me if you spot a mistake.

3.2 Decoherence and measurements

With the tools provided so far, we can now begin our first discussion of how decoherence arises. There will be a strong conceptual link with "measurements", so let's discuss those first.

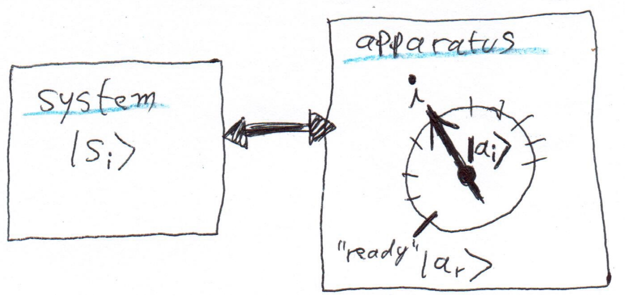
3.2.1 Ideal von Neumann measurements

The postulates in section 1.5.1 attribute a special status to the act of a measurement, that does not seem justified from fundamental principles. We now develop a line of thought due to von Neumann, that attempts to treat the measurement apparatus used to observe a quantum system in a quantum mechanical fashion as well.

Suppose the quantum system $\mathcal{S}$ to be observed has a basis $\{|s_n\rangle\}$. For the apparatus $\mathcal{A}$ we assume a basis $\{|a_n\rangle\}$. Let one of those basis states represent the initial state of the apparatus before it has done any measurement, called "ready state" $|a_r\rangle$. Now suppose the system is in a specific one of the basis states, say $|s_i\rangle$ initially. The act of measurement then must correspond to the unitary evolution

$$|s_i\rangle \otimes |a_r\rangle \rightarrow |s_i\rangle \otimes |a_i\rangle, \tag{3.20}$$

where the apparatus has made a transition to a state $|a_i\rangle$ indicating that it has measured $|s_i\rangle$ for the system. This is sketched in the figure below, adapted from SD. See (2.14) for the "definition" of "$\rightarrow$".



left: von Neumann scheme for an ideal quantum measurement.

- We assume here that each state of the system $|s_i\rangle$ results in the apparatus to take a distinguishable state $|a_i\rangle$ with 100% probability. In this sense the measurement is assumed perfect.

- Note that in (3.20), after the measurement, the system is still in $|\, s_i \,\rangle$. Such a scheme is called quantum non-demolition (QND) measurement. For the Stern-Gerlach apparatus reviewed in section 3.1.2 to make a QND measurement *before* atoms hit the screen, we need to have a non-destructive way to infer which beam the atoms are in, for example by weakly scattering light off them. *Once atoms hit the screen*, the scheme is no longer QND, since the screen may have changed the spin-state.

Now the quantum system will in general be in a superposition of its basis states $|\, \psi \,\rangle = \sum_n c_n |\, s_n \,\rangle$. Since the TDSE (1.8) is linear, we can then infer the complete

Von Neumann Measurement Evolution

$$|\, \psi \,\rangle \otimes |\, a_r \,\rangle = \left(\sum_n c_n |\, s_n \,\rangle \right) \otimes |\, a_r \,\rangle \to \sum_n c_n \big(|\, s_n \,\rangle \otimes |\, a_n \,\rangle \big), \tag{3.21}$$

- Note the initial state in (3.21) is separable, the final one entangled (see section 1.5.4). Entanglement has thus been created dynamically.
- For a specific example of unitary dynamics implied by "$\to$" in (3.21), see the spin-boson evolution in section 2.2.1 and assignment 1. There, the harmonic oscillator can be thought of as having "measured" the spin, with the left-hand-side (right-hand-side) coherent states corresponding to the $|\, a_i \,\rangle$, that are taken up depending on whether the sytem was in $|\, s_i \,\rangle \in \{|\uparrow\rangle, |\downarrow\rangle\}$.
- Depending on how "macroscopic" we have assumed our measurement device to be, we would develop the same conceptual worries with the state (3.21) as in the Schrödingers cat example, section 1.1 (it is in fact the same type of state). We might still be OK with the idea, when the apparatus $\mathcal{A}$ is say a single quantum oscillator, maybe a nano-mechanical spring, whereas if $\mathcal{A}$ was an oscilloscope, our intuition would reject it.

The conceptually very simple line Eq. (3.21) now allows us to clearly state three major problems with the concept of a quantum measurement. Some of these can be resolved by decoherence, but some others cannot.

3.2.2 Measurement Problems

I: The preferred basis problem

Suppose we have done our ideal van-Neumann measurement and wish to conclude from the entanglement structure in the final state of Eq. (3.21), that the apparatus has indeed "measured" which state of the specific system basis $|\, s_n \,\rangle$ the system "was in". I.e. after some still not quantum mechanically described collapse, the system is in state $|\, s_n \,\rangle$ with probability $|c_n|^2$.

However we can pick an arbitrary other basis for the apparatus, say $|\, a'_n \,\rangle$, express our former basis as $|\, a_n \,\rangle = \sum_k \langle\, a'_k \,|\, a_n \,\rangle |\, a'_k \,\rangle$, and then rewrite the final state in (3.21) as (exercise)

$$\sum_n c_n \big(|\, s_n \,\rangle \otimes |\, a_n \,\rangle \big) = \sum_k d_k \big(|\, s'_k \,\rangle \otimes |\, a'_k \,\rangle \big), \tag{3.22}$$

with new system states $d_k |\, s'_k \,\rangle = \sum_n c_n |\, s_n \,\rangle \langle\, a'_k \,|\, a_n \,\rangle$, where d_k normalises the state $|\, s'_k \,\rangle$.

- Since mathematically after the "quantum-measurement" above the discussed bases are equivalent, the question is: What picks the preferred basis in terms of which we obtain results after wave-function collapse?

Example for measurement problem: The problem is particularly severe, if the coefficients c_n are such that also the $|\, s'_n \,\rangle$ form an orthogonal basis of the system. Let's consider the Stern-Gerlach example (section 3.1.2) again, supposedly measuring a system (gold atom) in the state $|\, \psi \,\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. According to (3.21), the system+apparatus will evolve into $|\, \Psi \,\rangle = (|\uparrow\rangle |\, a_\uparrow \,\rangle + |\downarrow\rangle |\, a_\downarrow \,\rangle)/\sqrt{2}$, where the apparatus state $|\, a_\uparrow \,\rangle$ can be thought of as the position wave-function for the atom having moved into the upper beam due to the spin-dependent force felt in the magnetic field (see also assignment 1 for spin-dependent force).
However following (3.22) we could also write $|\, \Psi \,\rangle = (|\leftarrow\rangle |\, a_\leftarrow \,\rangle + |\rightarrow\rangle |\, a_\rightarrow \,\rangle)/\sqrt{2}$, where $|\, a_\leftarrow \,\rangle = (|\, a_\uparrow \,\rangle + |\, a_\downarrow \,\rangle)/\sqrt{2}$ and $|\, a_\rightarrow \,\rangle = (|\, a_\uparrow \,\rangle - |\, a_\downarrow \,\rangle)/\sqrt{2}$ (exercise). Thus simply from the entanglement-structure of the system+apparatus state, we could argue that the machine has measured both non-commuting observables $\hat{S}_x$, $\hat{S}_z$, which cannot be possible.
In fact we know that the apparatus would have to be designed differently if we wanted to measure $\hat{S}_x$, namely with magnetic field oriented along the x-axis instead of the z-axis.

II: The problem of Nonobservability of Interference

- The entangling dynamics of the kind (3.21) generically arises whenever a quantum system evolves while interacting with an environment. This suggests that entangled and superposition many-body states should be omnipresent also in the macroscopic world, since everything more or less interacts with something else. However, quantum mechanics is clearly not apparent in the large majority of macroscopic phenomena around us. In other words, we typically do not observe interference effects with macroscopic objects. Explaining this is the "problem of Nonobservability of Interference".

III: The problem of Outcomes

- The final question is, after having arrived at the superposition state (3.21) due to the unitary dynamics describing the measurement apparatus, why do we not actually "experience" that superposition? Why do we instead find a specific outcome and what selects which of the possible outcomes we find?
- This is actually an unresolved problem (yours), decoherence cannnot provide any answer for that.
- A possible approach have been proposal to extend quantum mechanics by a "physical collapse model", i.e. a mechanism that creates the otherwise only postulated wave function collapse. Other ideas embrace macroscopic superpositions in more philosophical ways. We will try to survey some of these ideas in the last week(s) of the lecture, time permitting.

3.2.3 Environmental monitoring and Decoherence

After having pointed out three major problems with the concept of a quantum-measurement, let us proceed to their partial resolution by the concept of decoherence through "environmental monitoring". The latter term is coined, since it turns out that the evolution in a von Neumann measurement (3.21) actually has the same structure as the evolution of a quantum system in contact with an environment, see e.g. (2.14). Thus if we consider (3.21) to be the evolution of a measurement apparatus "monitoring" our quantum system in state $|\,\psi\,\rangle = \sum_n c_n|\,s_n\,\rangle$, we have to consider the generalisation of (3.21) into

Entangling System-Environment Evolution

$$|\,\Psi(0)\,\rangle = |\,\psi\,\rangle \otimes |\,E(0)\,\rangle = \left(\sum_n c_n|\,s_n\,\rangle\right) \otimes |\,E(0)\,\rangle \rightarrow |\,\Psi(t)\,\rangle = \sum_n c_n\big(|\,s_n\,\rangle \otimes |\,E_n(t)\,\rangle\big), \tag{3.23}$$

as representing monitoring of the system by the environment.[a] We have adapted (3.23) to the example preceding (2.14): $|\,E(0)\,\rangle$ is the initial state of the environment (there oscillator), with a total state not entangled with the system. As time t goes on, the state evolves into one that is more and more entangled with the system, with the environment evolving into state $|\,E_n(t)\,\rangle$ if the system was in $|\,s_n\,\rangle$, (in the example the $|\,E_n(t)\,\rangle$ were the two *different* coherent states of the oscillator).

[a] We distinguish the total state $|\,\Psi\,\rangle$ from the system state $|\,\psi\,\rangle$.

- Note that (3.23) does not contain any non-trivial evolution within system and environment separately, only that due to interactions. However a component (3.23) will always be part of any coupled evolution.

The just discovered formal equivalence of system-environment and system-"measurement apparatus" evolution, motivates the definition of the

Measurement limit of interactions /quantum measurement limit, as the case where for system-environment models as discussed in section 2, we can neglect all Hamiltonians except $\hat{H}_{\rm int}$. Thus the evolution is entirely dominated by the system-environment interactions.

We can now use the simple mapping (3.23) to *qualitatively* resolve 2 of the 3 problems outlined earlier. For a *quantiative* resolution, await chapter 4.

The reason for Non-observability of Interference (resolving measurement problem II)

The entangling evolution (3.23) ought to be unavoidable and generic for any insufficiently isolated quantum system, suggesting many-body entangled states are everywhere. However assuming we can only observe the system itself, precisely this can be the reason why we do not observe macroscopic quantum effects, as seen already in section 3.1.4:

Since we can only observe the system, all observables follow from its reduced density matrix, which for the final state in (3.23) is (see calculation in (3.19))

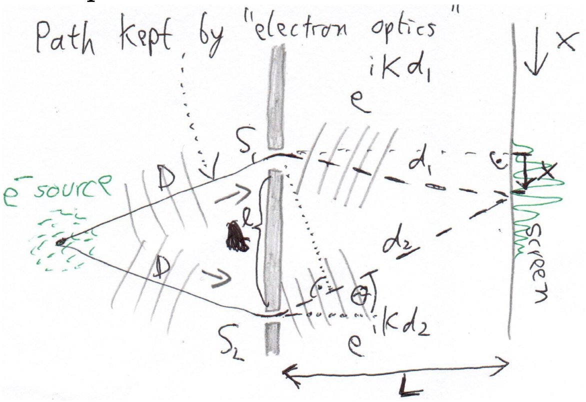
$$\hat{\rho}_S = \frac{1}{2}\Big(|\, s_1 \,\rangle\langle\, s_1 \,| + |\, s_2 \,\rangle\langle\, s_2 \,| + |\, s_1 \,\rangle\langle\, s_2 \,| \langle\, E_2(t) \,|\, E_1(t) \,\rangle + |\, s_2 \,\rangle\langle\, s_1 \,| \langle\, E_1(t) \,|\, E_2(t) \,\rangle \Big). \tag{3.24}$$

- We have reduced the number of system states to two again and used $c_1 = c_2 = 1/\sqrt{2}$ in (3.23) for simplicity.

Once $\langle\, E_1(t) \,|\, E_2(t) \,\rangle \approx 0$, we will see no interference in the system anymore. The time by which this happens typically is very short for a macroscopic environment, as we see in examples later. Let us first consider an explicit example discussing "interference fringes" :

Example: Decoherence in the matter-wave double slit experiment: Let us consider the matter-wave version of the double slit experiment in section 1.3, sketched below. After the region of the slits, we can write the total electron matter wave roughly as $\psi(\mathbf{r}) = \frac{1}{\sqrt{2}}\mathcal{N}\left(\exp\left[ikd_1\right] + \exp\left[ikd_2\right]\right)$, with coordinates indicated in the sketch. The electron probability on the screen is then $|\psi(\mathbf{r})|^2 \approx \mathcal{N}^2\big(1 + \cos\left(k(d_1 - d_2)\right)\big)$. Rewriting in terms of angle on the screen we obtain $|\psi(\theta)|^2 \approx \mathcal{N}^2\big(1 + \cos\left(kl\theta\right)\big)$, as following from the result for (1.4) in optics as well.

Example continued:



left: Sketch of the matter-wave double slit experiment, all essential features are as with the optical setup, even though practical challenges are vastly different.

Now assume the electron might have interacted with the slit material at the moment of its crossing. This interaction will be microscopically different in the left and right slit. After passing the slit, we then assume a total state $|\,\Psi(\mathbf{r})\,\rangle = \frac{1}{\sqrt{2}}\mathcal{N}\left(\exp\left[ikd_1\right]|\,L\,\rangle + \exp\left[ikd_2\right]|\,R\,\rangle\right)$, where we write the electron state in the position representation and the (abstract) state of the slit material as $|\,L/R\,\rangle$, depending on whether the electron has gone through the **L**eft or **R**ight slit.

Since our screen does not measure the slit state, we have to now utilize the reduced density matrix for the electron, obtained like in (3.19), (3.24). Let us first express the state above in terms of position x on the screen, using $d_1 \approx L + x^2/(2L)$, $d_2 \approx L + (l-x)^2/(2L)$, to find $|\,\Psi(x)\,\rangle = \frac{\mathcal{N}e^{ikL}}{\sqrt{2}}\left(\exp\left[ikx^2/(2L)\right]|\,L\,\rangle + \exp\left[ik(l-x)^2/(2L)\right]|\,R\,\rangle\right)$, and hence

$$\hat{\rho}(x,x') = |\,\Psi(x)\,\rangle\langle\,\Psi(x')\,| = \frac{\mathcal{N}^2}{2}\Bigg(e^{ik\frac{(x^2-x'^2)}{2L}}|\,L\,\rangle\langle\,L\,| + e^{ik\frac{((l-x)^2-(l-x')^2)}{2L}}|\,R\,\rangle\langle\,R\,|$$
$$+ e^{ik\frac{(x^2-(l-x')^2)}{2L}}|\,L\,\rangle\langle\,R\,| + e^{ik\frac{((l-x)^2-x'^2)}{2L}}|\,R\,\rangle\langle\,L\,|\Bigg) \tag{3.25}$$

resulting in

$$\hat{\rho}_S(x,x') = \frac{\mathcal{N}^2}{2}\Bigg(e^{ik\frac{(x^2-x'^2)}{2L}} + e^{ik\frac{((l-x)^2-(l-x')^2)}{2L}}$$
$$+ e^{ik\frac{(x^2-(l-x')^2)}{2L}}\langle\,R\,|\,L\,\rangle + e^{ik\frac{((l-x)^2-x'^2)}{2L}}\langle\,L\,|\,R\,\rangle\Bigg). \tag{3.26}$$

For the electron probability on the screen, $\hat{\rho}_S(x,x)$, we obtain

$$\hat{\rho}_S(x,x) = \frac{\mathcal{N}^2}{2}\Big(2 + \exp\left[-ikl(x-l/2)/L\right]\langle\,R\,|\,L\,\rangle + \exp\left[ikl(x-l/2)/L\right]\langle\,L\,|\,R\,\rangle\Big). \tag{3.27}$$

indicating an unperturbed interference pattern only if $|\,L\,\rangle = |\,R\,\rangle$ (for example if the electron actually has not affected the screen and $|\,L\,\rangle = |\,R\,\rangle = |\,\phi_{\text{screen}}(t=0)\,\rangle$). If $\langle\,L\,|\,R\,\rangle = 0$, no interference is seen at all.

- We can identify $|\,L/R\,\rangle$ also with the states of some measurement device that allows us to infer which slit the electron has taken, leading to the result that whenever we or the environment obtain some "which-path-information", the interference pattern is gone.

- There are significant technical challenges in the matter-wave experiment compared to optics. For electron-optics we require relatively fast electrons, $E = 50$ keV in the first experiment [C. Jönsson Z. Phys. **161** 454 (1961), Am. J.Phys. **42** 4 (1974)], with a de-Broglie wavelength $\lambda = 0.05\mathring{A}$. Since this is much less than the size of an atom, we cannot actually make slits that are small against the wavelength, as we would in optics.

- See this movie of the above experiment based on single electrons [A. Tonomura *et al.* Am. J. Phys. **57** 117 (1989)]. This version of the experiment is one of the most paradigmatic demonstrations of particle-wave duality.

- One can nowadays perform such experiments with complex particles such as bio-molecules [Gerlich *et al.* Nat. Comm. **2** 263 (2011)]. While for electrons decoherence as described above can be made small, for these systems it cannot. The next plans, are to do it with a virus.

Another element that is included in (3.24) through the time-dependence of $E_n(t)$, is the slow loss of coherence over some characteristic decoherence time. For this we can make use of our earlier spin-boson model example, see section 2.2.1 and assignment 1:

Example: Dynamical decoherence in the spin-boson model: Let us go back to the example in section 2.2.1 of a spin interacting with an oscillator in the simplified spin-boson model. We determine the reduced density matrix for the spin only, applying (3.24) to (2.13) and obtain

$$\hat{\rho}_S = \frac{1}{2}\bigg(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\downarrow|\langle\,\alpha_{Q+}\,|\,\alpha_{Q-}\,\rangle + |\downarrow\rangle\langle\uparrow|\langle\,\alpha_{Q-}\,|\,\alpha_{Q+}\,\rangle\bigg). \tag{3.28}$$

Using Eq. (1.26), we can calculate the overlap of two coherent states as $\langle\,\alpha\,|\,\beta\,\rangle = e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2-2\beta^*\alpha)}$. In assignment 1 you found $\alpha_{Q+} = -\alpha_{Q-} = \bar{\kappa}/\omega(1-e^{i\omega t})$, We find $\langle\,\alpha_{Q+}\,|\,\alpha_{Q-}\,\rangle = e^{4\frac{\kappa^2}{\omega^2}(\cos[\omega t]-1)}$, which for short times becomes

$$\langle\,\alpha_{Q+}\,|\,\alpha_{Q-}\,\rangle = e^{-2\kappa^2 t^2}. \tag{3.29}$$

The spin thus has "decohered" (precluding the visibility of interferences), after a time scale $\Gamma_{\text{decoh}} \sim \kappa^{-1}$.

- We can make the decoherence dynamics even more explicit by calculating the time dependent Purity $P(t)$, using Eq. (3.10), from (3.28).

- In the preceding example, coherence / purity would periodically be restored after intervals $T = 2\pi/\omega$. This is because the single oscillator that we coupled the spin to is not really a large environment. Repeating the calculation with a large and larger number of oscillators, we see that the time of "revival" where $P(t) = 1$ again becomes later and later, until it becomes irrelevant.

- In the preceding example we are fortunate to be able to just calculate the dynamics of the entire setup (system+environment), and then form the reduced DM for the system. In general this will not be possible. In chapter 4 we learn how to avoid the detour of calculating the entire setup if we anyway care only about the system, and still quantitatively model the de-coherence dynamics, including the decoherence time.

- Importantly, the superposition character is not lost in the many-body state (3.23). It has in some sense "moved" from within the system only to the combination system+environment. We thus can no longer observe it, if we are confined to measuring the system only, which is fully described by (3.24). If we can make a sophisticated measurement on system+environment, we could still hope to demonstrate the quantum superposition character of (3.23). In all relevant cases, this is not possible.

Example: Decoherence in a spin-spin model: Finally consider a genuine many-component spin-spin model as in section 2.3, where we assume all dynamics is dominated by the system environment interaction (2.22) and neglect the other two Hamiltonians. This allows us to determine the full time evolution for an arbitrary number of spins.
We start from an initial state $|\,\Psi(0)\,\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes \sum_{s_1,s_2,\ldots} c_{s_1,s_2,\ldots}|\,s_1,s_2,\ldots\rangle$, where $s_i \in \{+1/2,-1/2\}$ and the part before $\otimes$ is the initial state of the system, the rest that of the spin-environment. You find (exercise/ SD 2.10.)

$$|\,\Psi(0)\,\rangle = a|\uparrow\rangle \otimes |\,\mathcal{E}_0(t)\,\rangle + b|\downarrow\rangle \otimes |\,\mathcal{E}_1(t)\,\rangle, \tag{3.30}$$

with $|\,\mathcal{E}_0(t)\,\rangle = |\,\mathcal{E}_1(-t)\,\rangle = \sum_{s_1,s_2,\ldots} c_{s_1,s_2,\ldots} e^{-i\left(\sum_n^N \kappa_n(-1)^{[1/2-s_n]}\right)t/\hbar}|\,s_1,s_2,\ldots\rangle$. We have seen this entanglement structure multiple times before.
For the system spin we will find $\hat{\rho}_S = \Big(|a|^2|\uparrow\rangle\langle\uparrow| + |b|^2|\downarrow\rangle\langle\downarrow| + ab^*r(t)|\uparrow\rangle\langle\downarrow| + a^*br^*(t)|\downarrow\rangle\langle\uparrow|\Big)$, with decoherence factor $r(t) = \sum_{s_1,s_2,\ldots} |c_{s_1,s_2,\ldots}|^2 e^{-2i\left(\sum_n^N \kappa_n(-1)^{[1/2-s_n]}\right)t/\hbar}$.
It is possible to make some statistical arguments for randomly oriented environmental spins, to show that $r(t)$ scales as $r(t) \sim 2^{-N}$ with the number of spins, and as $r(t) \sim 2^{-\Gamma^2 t^2}$ with time.

- The main purpose of this example, is to show the generic feature, that coherence is suppressed exponentially with increasing size N of the environment.

3.2.4 Pointer states and Environmental Superselection

Earlier we had argued that entangling system-environment evolution such as (3.21) is the *generic case*[4]. However it does not happen for all initial states, as shown in the following example.

[4]That means, take a random initial state, almost always this will be part of the dynamics.

Example: No decoherence in the spin-boson model: Now consider the spin-boson model with a different initial state than considered in the example of section 2.2.1 or assignment 1. However we still begin in a superposition state $|\,\Psi(0)\,\rangle = \frac{1}{\sqrt{2}}(|\leftarrow\rangle + |\rightarrow\rangle) \otimes |\,0\,\rangle$. Since we can express this as $|\,\Psi(0)\,\rangle = |\uparrow\rangle \otimes |\,0\,\rangle$ and Schrödinger evolution is linear, we can read off the final state from our earlier results as $|\,\Psi(0)\,\rangle = |\uparrow\rangle \otimes |\,\alpha_{Q-}\,\rangle$. Here the system *has not* entangled itself with the environment. Consequently upon forming the reduced density matrix of the system $\hat{\rho}_S = |\uparrow\rangle\langle\uparrow| = \frac{1}{2}\Big(|\leftarrow\rangle\langle\leftarrow| + |\rightarrow\rangle\langle\rightarrow| + |\leftarrow\rangle\langle\rightarrow| + |\rightarrow\rangle\langle\leftarrow|\Big)$, which has fully preserved the initial coherence/ superposition despite the interaction with the environment.

This tells us that in the presence of an environment, not all bases of the system Hilbertspace are equivalent, in the sense that superpositions expressed in certain bases will decohere, while in other bases they may not. We define

Pointer states as the preferred states of the system in contact with a certain environment. This means those states which during evolution give rise to least entanglement with the environment.

We can find the pointer states in the quantum measurement limit (see end of section 3.2.3). In this case only $\hat{H}_{\text{int}}$ is relevant. We demand that an initial product of some system and some environment state $|\,\Psi(0)\,\rangle = |\,s_i\,\rangle|\,E_0\,\rangle$ remains in product form under the action of the Hamiltonian

$$|\,\Psi(t)\,\rangle = e^{-i\hat{H}_{\text{int}}t/\hbar}|\,s_i\,\rangle|\,E_0\,\rangle \stackrel{!}{=} \lambda_i|\,s_i\,\rangle e^{-i\hat{H}_{\text{int}}t/\hbar}|\,E_0\,\rangle = |\,s_i\,\rangle|\,E_i(t)\,\rangle. \tag{3.31}$$

We can see that (3.31) is fulfilled if $|\,s_i\,\rangle$ is an

Eigenstate of the system-part of the interaction Hamiltonian: By this we mean that

$$\hat{H}_{\text{int}}|\,s_i\,\rangle|\,E\,\rangle = \alpha_i|\,s_i\,\rangle|\,E'\,\rangle. \tag{3.32}$$

for arbitrary environment state $|\,E\,\rangle$. Then $|\,E'\,\rangle$ is *some*, typically different environment state.

While it does not always have to be the case, in all the examples of section 2 we have the form

$$\hat{H}_{\text{int}} = \hat{O}_S \otimes \hat{O}_{\mathcal{E}}, \tag{3.33}$$

with $\hat{O}_S$ acting only on the system and $\hat{O}_{\mathcal{E}}$ only on the environment. In that case, the pointer states are simply the eigenstates of $\hat{O}_S$.

Example: Pointer states of spin-boson model: Using the discussion above, we see that the pointer states of the spin-boson model, with interaction term $\hat{H}_{\text{int}} = \hat{\sigma}_z \otimes \sum_i \bar{\kappa}_i \left(\hat{a}_i + \hat{a}_i^\dagger\right)$ are eigenstates of $\hat{O}_S = \hat{\sigma}_z$, hence $|\uparrow\rangle$ and $|\downarrow\rangle$.
This matches our experience in earlier examples, where we saw that $\hat{H}_{\text{int}}$ evolves an initial state $|\leftarrow\rangle \otimes |0\rangle$ into an entangled state, while it leaves $|\uparrow\rangle \otimes |0\rangle$ as a product.

- In general superpositions of pointer states will not be pointer states.
- However in some cases this can happen, then we talk about a "pointer subspace" or "decoherence free subspace".
- The singling out of a preferred basis by the system-environment interaction also has been given the name environment induced superselection or "ein-selection".

Preferred basis of a measurement apparatus (resolving measurement problem I)

We can resolve the preferred basis problem, by applying the concept of pointer states to a measurement apparatus, which is measuring a quantum system, while the apparatus in turn is in contact with an environment, Recall that we found in (3.22) that subsequent to a von Neumann measurement, there are multiple ways expressing the entangled state that would seem to indicate that our apparatus has measured lots of non-commuting observables at once:

$$\sum_n c_n \big(|s_n\rangle \otimes |a_n\rangle\big) = \sum_k d_k \big(|s'_k\rangle \otimes |a'_k\rangle\big). \tag{3.34}$$

Example: Pointer states of Stern-Gerlach apparatus: If we return to the example given after (3.22), of a Stern-Gerlach apparatus measuring a spin, we found the competing bases of the apparatus $\{|a_\uparrow\rangle, |a_\downarrow\rangle\}$ versus $\{|a_\leftarrow\rangle, |a_\rightarrow\rangle\}$, where we had said $|a_\uparrow\rangle$" = " atom went to upper beam ($|a_\downarrow\rangle$" = " atom went to lower beam). The distinction between upper and lower beam is in terms of the position operator $\hat{x}$.
Now consider that the atom is always in contact with some surrounding environment (let it be black-body radiation or vacuum imperfections, i.e. other atoms floating around). This environment may be too complicated to fully tackle, but what we can tell, is that the system-environment interaction will be mainly a function of the position of the atom $H_{\text{int}} = \hat{f}[\hat{x}]$. Thus pointer states are position eigenstates[a]. This now singles out our measurement basis $\{|a_\uparrow\rangle, |a_\downarrow\rangle\}$ (which are position eigenstates) from $\{|a_\leftarrow\rangle, |a_\rightarrow\rangle\}$, which both are superpositions of the atom being in the upper and lower beam simultaneously. The latter would immediately decohere in contact with the environment so that the superposition (3.34) ceases to exist.

[a] If you find these pathological, consider a very strongly localized Gaussian wavepacket.

We thus find the

Resolution of the preferred basis problem A measurement apparatus measures that basis $\{|\, s_n \rangle\}$ of the system, which after the measurement has evolved into an entangled state of the kind (3.34) involving the pointer states $\{|\, a_n \rangle\}$ of the apparatus.

We still have a problem with outcomes (not resolving measurement problem III)

After all the $|\, E_n(t) \rangle$ in Eq. (3.23) have become orthogonal, the reduced density matrix for the system will be

$$\hat{\rho}_S = \sum_n |c_n|^2 |\, s_n \rangle\langle\, s_n\,|. \tag{3.35}$$

This correctly describes the *measurement statistics* of a large number of repeat measurements on an identical system, giving a chance $|c_n|^2$ to find the eigenvalue for $|\, s_n \rangle$, the system being in $|\, s_n \rangle$ subsequently.

It still does not describe in any more satisfactory way than (non-open) quantum mechanics, why we do not measure some effect of all the components n in a single measurement, but instead only one of the n is selected as the outcome and the state subsequently changed to $|\, s_n \rangle$.