Week 11

PHY 435 / 635 Decoherence and Open Quantum Systems
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7 Extensions of Quantum Mechanics

We had seen since section 3.2, that the theory of decoherence in the framework of open quantum system does help a lot in addressing the "measurement problems" discussed in section 3.2. Note that everything we covered was strictly in the framework the usual quantum mechanics. We had also already stated that decoherence theory does *not* really offer a solution to the dissatisfactory need to *postulate* the collapse of the wave function and hence the existence of definite outcomes of measurements in quantum theory [measurement problem III, problem of outcomes].

We now briefly mention a few speculative[7] ideas in which the collapse problem is addressed *within the mathematical framework*. This means that the proposals actually attempt to change the laws of quantum mechanics, rather than just their interpretation. These changes are tightly constrained, since they must not contradict the multitude of successful and accurate verifications of quantum mechanics.

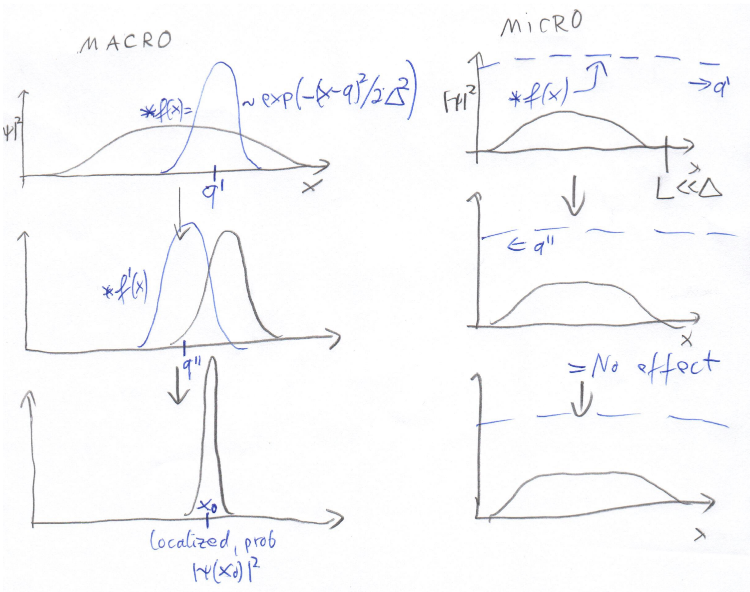
7.1 Physical collapse models

The final result of the von Neumann measurement in the presence of decoherence was $\hat{\rho}_S(t_f) = \sum_n |c_n|^2 |\, s_n \rangle | \, a_n \rangle \langle \, s_n \, | \langle \, a_n \, |$, see Eq. (3.35) and Eq. (3.21). Recall that chosen basis $\langle \, a_n \, |$ had to be the pointer basis of the apparatus in the presence of the environment.

Collapse models try to augment the measurement process, by changing the time evolution of $\hat{\rho}_S$ (or $| \, \Psi \, \rangle$), such that the final state is actually only $\hat{\rho}_S(t_f) = | \, s_k \rangle | \, a_k \rangle \langle \, s_k \, | \langle \, a_k \, |$, for one specific k. However the k has to be random such that it occurs with probability $|c_k|^2$ (defined by the initial state). To this end, different theories modify the Schrödinger equation by (i) random noise terms, turning it into a stochastic differential equation (SDE), (ii) non-linear terms[8] or the occasional random projection onto more localised wavefunctions. The latter process is sketched in the figure below.

[7] This means that to our knowledge there were no experiments that support these ideas compared to others.

[8] Note that the entire von-Neumann measurement chain relies on the *linearity* of the SE, so non-linearities are a logical way out.



left: continuous spontaneous localisation Sketch of wave function evolution in the continous spontanous localisation model. Spatially delocalized states on scales $x \ll \Delta$ (microscopic ones) are not much affected so that the usual QM works there. Macroscopically delocalized wavefunctions will localize dynamically in this way.

One motivation for interference experiments with larger and larger molecules (^{60}C, Biomolecules) such as mentioned in section 3.2.3, is to verify or exclude such collapse models. It turns out that this is very hard, since the special predictions of these are very similar to the effects of decoherence. So the interfering buckyball has to be exceptionally well shielded form decoherence (possible in principle but hard in practice) to see the effect of spontaneous localisation (which would be un-avoidable if it exists).

For example a master equation can also be derived for continuous spontaneous localisation, and then annoyingly has the same form as Eq. (4.47), with a term $\frac{d}{dt}\rho(x,x') \sim -\Lambda(x-x')^2\rho(x,x')$ that destroys spatial coherences. The advantage of collapse models though, is that in the end the sytem really is only in one location, not in a superposition state.

Further reading: "Dynamical reduction models", A. Bassia and G. Ghirardi, Phys. Rep. **379** 257 (2003).

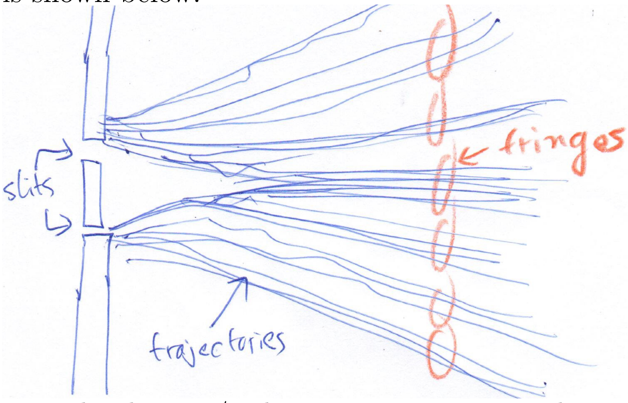
7.2 Bohmian Mechanics

In Bohmian mechanics, there actually *are* particles, that are only guided by their matter wave. We assume the position of the particle is $\mathbf{q}$. It will then move according to

$$\frac{d\mathbf{q}}{dt} = \mathbf{v} = \frac{1}{m}\mathrm{Im}\left(\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi|^2}\right)(\mathbf{q}), \tag{7.1}$$

where Ψ is the usual wave function that obeys Schrödinger's equation. The construction makes sure that experimental observations regarding the probabilities of particle positions would be consistent with the usual quantum mechanics. Compare (7.1) with the hydro-dynamics formulation of quantum mechanics (density, velocity).

The difference to the usual interpretation is, that the particle initially *does have* a fixed position $\mathbf{q}(t=0)$, which is however unknown. We could use an equation such as Eq. (7.1) for particles with lots of different random initial positions passing through a double slit, and draw their tracks. This is shown below:



left: Double slit interference in Bohmian Mechanics For a large number of initial conditions, we realize the usual double slit inteference pattern as regions with many ending trajectories (orange). Following an individual trajectory shows some irritating non-Newtonian behavior. See also SD for better picture.

Particle physics / relativistic quantum mechanics actually requires the concept of quantum fields rather than particles. Quantum fields are a more advanced version of a wave function. The requirements arise from elementary conditions such as causality and Lorentz invariance. For these reasons a forced return to the particle concept as above is not very popular.

Since Bohmian Mechanics reproduces quantum mechanics through the assumption of precise but unknown initial particle positions, it is an example of a *hidden variable theory*. These attribute all the randomness of quantum mechanics to unknown variables (i.e. we simply don't know the real, deterministic, theory yet).

7.3 Many Worlds theory

We can continue the von-Neumann chain, by including ourselves as the observer into the superposition:

$$|\,\psi\,\rangle \otimes |\,a_r\,\rangle \otimes |\,b_r\,\rangle = \left(\sum_n c_n|\,s_n\,\rangle\right) \otimes |\,a_r\,\rangle|\,b_r\,\rangle \to \sum_n c_n\big(|\,s_n\,\rangle \otimes |\,a_n\,\rangle|\,b_n\,\rangle\big), \tag{7.2}$$

Here $|\,b_r\,\rangle$ denotes the state of your brain when you are ready to do the experiment and $|\,b_n\,\rangle$ that after you have seen that the apparatus has indicated $|\,a_n\,\rangle$.

Many worlds theory tries to embrace that simply all components of the superposition in fact happen, only since we are part of one branch we cannot experience the other branches. Essentially, whenever any quantum evolution takes place, the universe splits up into further universes (multiverse). Each possible result of a measurement would happen in some of them.

It is clear from the discussion that this now more to do with philosophy than physics, and up to you which viewpoint (e.g. many worlds or problem of outcomes) you find more irritating. Since we can per definition not make experiments in another universe, the idea is also difficult to disprove.

As an intermediate step, some of the early researchers on quantum theory had speculated that it is our consciousness $|\,b_n\,\rangle$, which ultimately collapses the superposition.