PHY635, II-Semester 2022/23, Assignment 5

Instructor: Sebastian Wüster
Due-date: Email by 31.3.2023

(1) Electron gas in 2D: We can now routinely create materials or devices in which electron dynamics is effectively constrained to two dimensions, such as in graphene or layered semi-conductor stuctures.

(a) For a gas of electrons free to move in only 2 dimensions, find the Fermi energy and temperature in terms of the 2D electron density. [*5 points*]

(b) Assume the 2D gas electrons are confined in a semi-conductor heterostructure, with the middle layer (containing the electron gas) composed of GaAs. Find or google a simple estimate for the 2D electron density in this system and use this to evaluate the quantities calculated in (a) for this system. [*5 points*]

(2) White dwarf stars: Derive the maximum mass that a white dwarf star can have, assuming electrons are ultra-relativistic (energy $\gg$ rest mass energy) and the density in the star is uniform. [*10 points*]

(3) Bosonic versus Fermionic ground-states
The template file `Assignment5_phy635_program_draft_v1.xmds` finds the ground-state of the Schrödinger equation for two Bosonic ^{7}Li atoms in a one-dimensional harmonic trap using imaginary time evolution. Lithium also has a long-lived Fermionic isotope ^{6}Li.

(4a) From the many-body wavefunction, derive an expression for the total density of atoms at position x. Implement the sampling of that in the last output block of the script provided. Note that the block is set up to integrate whatever is inserted over the coordinate x_2. [*2 points*].

(4b) Analytically show that the imaginary time (and real time) Schrödinger equation for two particles preserves Bosonic and Fermionic symmetries of the wave-function. [*1 points*]

(4c) Using (4b), modify the code such that it can find the corresponding ground-state for two Fermionic atoms. Compare total densities for the Fermionic and Bosonic cases with the scripts provided. How is the Fermionic density pattern called? [*3 points*]

(4d) Now assume the Bosons are interacting with a very short range but strong repulsive interaction $U(x_1 - x_2) = A\exp(-\frac{|x_1-x_2|^2}{2\sigma^2})$. Implement that into the imaginary time evolution, and compare the ground-state and total density for the interacting Bosons, with the non-interacting Fermions. Discuss. [*4 points*]