# PHY635, II-Semester 2022/23, Assignment 4

*Instructor: Sebastian Wüster*
*Due-date: Email by 17.3.2023*

**(1) Dark solitons:** Consider an infinitely extended homogenous BEC of atoms with mass $m$ in one-dimension[1] having a background-density $\rho$ and repulsive interactions with strength $U_0 > 0$ (no trap).

(a) Show that for the right choice of the parameter $\eta$ and the chemical potential $\mu$, the condensate wavefunction

$$\phi(x) = \sqrt{\rho}\tanh(x/\eta) \tag{1}$$

solves the TIGPE [4].
*Hints:* $\tanh(x)'' = -2\mathrm{sech}(x)^2\tanh(x)$, $\quad \mathrm{sech}^2(x) + \tanh^2(x) = 1$.

(b) Discuss the implications of the wavefunction in Eq. (1) and identify the physical meaning of the required $\eta$ [2].

(c) Your derivation in (a) had assumed a constant background density. Under which conditions do you think you could you still use your result in the centre of a very large Thomas-Fermi BEC? If you do this, you need a relation between atom-number, trapping parameters and peak density in the BEC. Find this relation for application in oder to later apply it in Q3(c) [2].

(d) Google the definition of the term "soliton" and discuss it in the context of your solution above [2].

**(2) Variational calculation of condensate width for weak interaction:** Consider a condensate with weak repulsive or attractive interactions $U_0$. Using the variational Ansatz:

$$\phi(x) = \sqrt{N}\frac{1}{(\pi\sigma(U_0)^2)^{1/4}}e^{-\frac{x^2}{2\sigma(U_0)^2}}, \tag{2}$$

find the optimal $\sigma(U_0)$ ($\sigma$ as a function of $U_0$) for either sign. Discuss your result [8].

**(3) Numerical condensate ground states and grey soliton dynamics:**
The script `Assignment4_phy635_program_draft_v2.xmds` first evolves the imaginary time GPE [Eq. (3.48)], for a certain duration of "imaginary time", followed by the real time GPE [Eq. (3.41)] for a second interval of "real time".

(a) Modify the code such that it includes a harmonic potential (use parameters provided), and give it the expected Thomas-Fermi profile for the chosen atom number and interaction constant as initial state (initial "guess" of the ground-state). You can use

[1] Ignoring the technicality that we shouldn't have one in 1D, in practice we can still have them in a finite system that really is 3D and just strongly confined in the other two directions.

`check_imagtime.m` to see if the imaginary time has converged (how? why does the script plot what it plots?) and to see how the true the ground-state looks like. Try to change parameters such that the result looks more (less) "Thomas-Fermi-like". Also change your initial state into some crazy choices. Discuss what you find. [4]

(b) Insert your dark soliton from Q1 at $x_0 = 0$ by uncommenting the filter-block provided and modifying its content. You can now use `density_slideshow_realttimeonly_v1.m` to test your answer from Q1. Discuss. [4]

(c) Finally, insert your solution from Q1 at $x_0 = 0$ and then change the complex phase of the condensate wavefunction on the RHS ($x > 0$) by $0.05\pi$, thus perturbing the system. Discuss what you find with `check_soliton_motion.m`. Discuss a few options how you could attempt to <u>analytically</u> understand your observations. [4]