PHY635, II-Semester 2022/23, Assignment 3

Instructor: Sebastian Wüster
Due-date: Email to TA by 10.2.2023

(1) Bose-Einstein condensation in different numbers of dimensions: Revisit the calculation of section 3.2.

(a) Then adapt it to a 2D and a 1D harmonic trap. Find the condensation temperature in either case. [5 pts]

(b) Next adapt it to a 3D equal side length infinite square well potential. [5 pts]

(c) Compare all dimensions and systems you have inspected and discuss the dependence of critical temperature on the number of dimensions and system details. [2 pts]

(2) Grand canonical ensemble: Consider a collection of non-interacting particles in the infinite square well potential $V(x)$ (zero between $x = 0$ and $x = a$, infinite outside). For whatever reasons[1], assume these to be in contact with an environment at temperature $T > T_{\rm crit}$ and also exchanging particles with it. Find the chemical potential required to have a given mean number N of particles in the box. [8 pts]

(3) Bose gas thermometry: Consider a partially condensed Bose gas of a mean number of $\bar{N} = 10^5$ ^{87}Rb atoms in an isotropic harmonic trap with trapping frequency $\omega = (2\pi)100$ Hz. Assume the atoms do not interact, because interactions are switched off using a Feshbach resonance, which we will discuss later.

(a) Find the specific heat $C = \partial E/\partial T$ of the Bose-gas above and below $T = T_{\rm crit}$, where E is the total energy of a Bose gas using the canonical ensemble. [2pts]

(b) Using a mathematica script or analytical calculations, find the atom numberdensity $\rho(r)$ as a function of radial distance from the centre of the trap r. Show plots of this for $T = 0$, $T = T_{\rm crit}/2$ and $T > T_{\rm crit}$. [5 pts]

(c) How do you propose to use this to measure the temperature of the Bose gas? [3 pts]

[1] theoretical ones