PHY 535/635 MBQM 2023 Assignment 1 solution

Question 1

(part a)

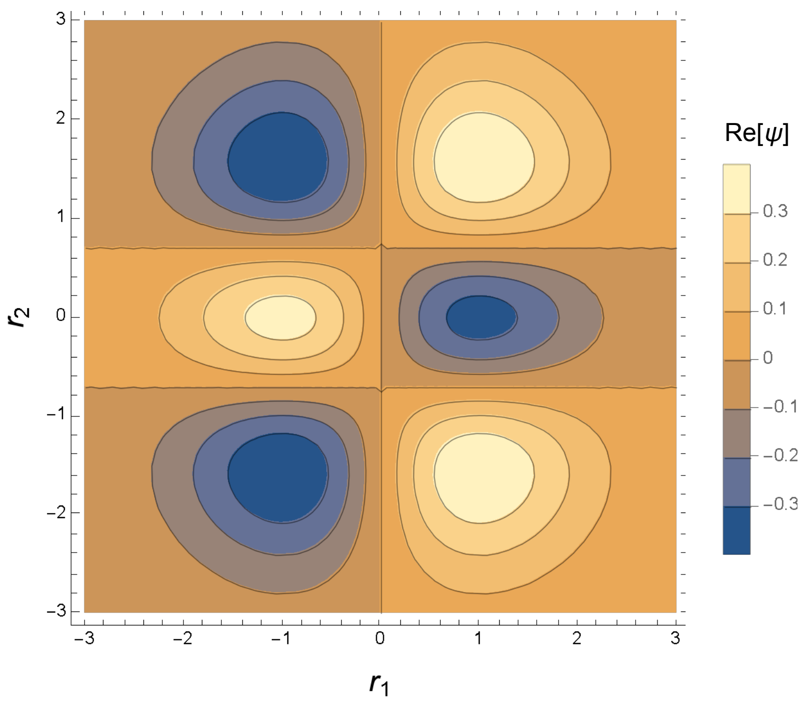
distinguishable: In this case we can write the product of the two proposed states without worrying about symmetrisations.

```
ϕsho[x_, n_] := (1/(Sqrt[2^n n!] (π σ^2)^(1/4))) HermiteH[n, x/σ] Exp[-x^2/2/σ^2]; σ := 1;
ψ[r1_, r2_] := ϕsho[r1, 1] × ϕsho[r2, 2];
```

```
In[5]:= Lplot = 3;
```

```
In[23]:= ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot},
  {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
  PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[23]=

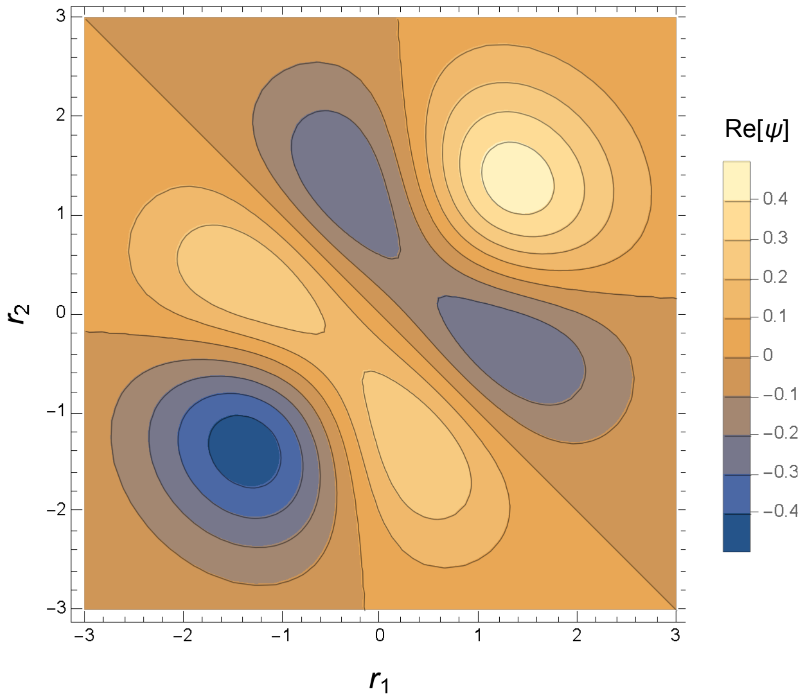


Bosons : For Bosons, we symmetrise the wavefunction

```
In[24]:= ψ[r1_, r2_] := (1/Sqrt[2]) (ϕsho[r1, 1] × ϕsho[r2, 2] + ϕsho[r1, 2] × ϕsho[r2, 1]);
```

In[25]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[25]=

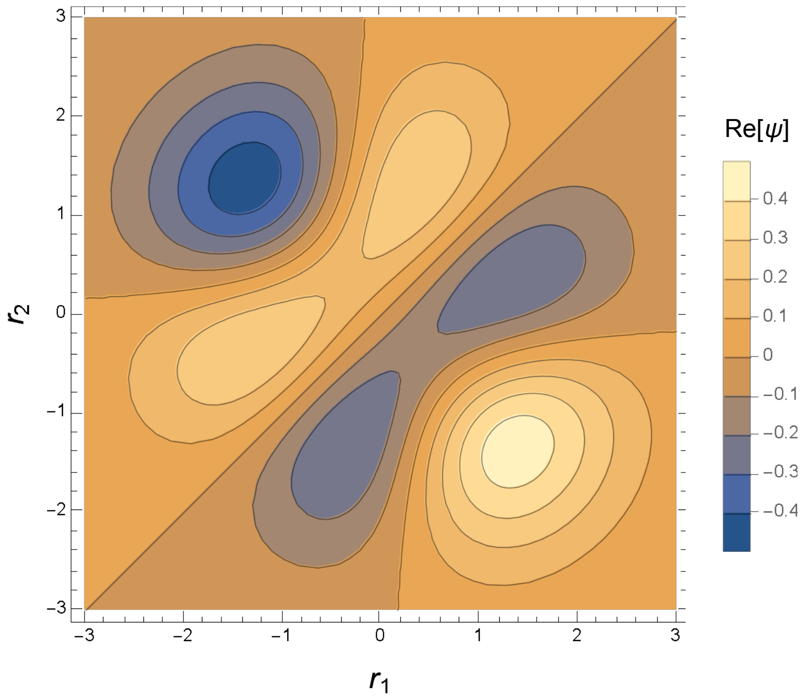


Fermions: For Fermions we anti-symmetrize

In[26]:=
```
ψ[r1_, r2_] := 1/Sqrt[2] (ϕsho[r1, 1] × ϕsho[r2, 2] - ϕsho[r1, 2] × ϕsho[r2, 1]);
```

In[27]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[27]=



We can see that all three options give different two - body wavefunctions .

(part b)

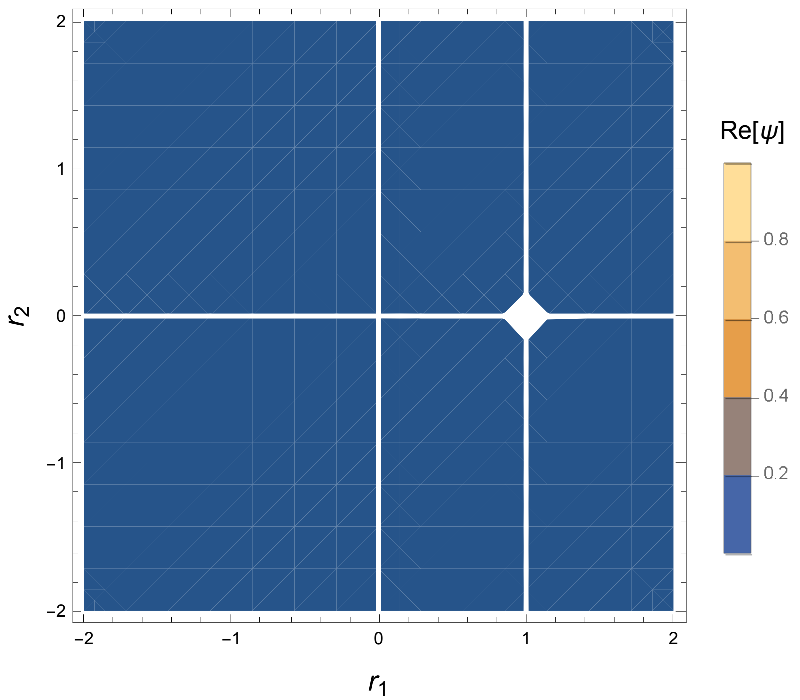
distinguishable: We take the ground state of neutron in the nucleus to be the ground state of the square well potential (see nuclear physics lecture). For the scattering particle we assume it does NOT penetrate into the nucleus, which we take from 0 to L.

```
ϕnuc[x_, n_] := If[x > 0 && x < L, Sqrt[2/L] Sin[n π x/L], 0]; L := 1;
kplot := 2 π / 0.3;
ϕscatt[x_, k_] := If[x < 0, (Exp[I k x] - Exp[-I k x]) / Sqrt[2 π], 0];
ψ[r1_, r2_] := v
```

In[54]:=
```
Lplot = 2;
```

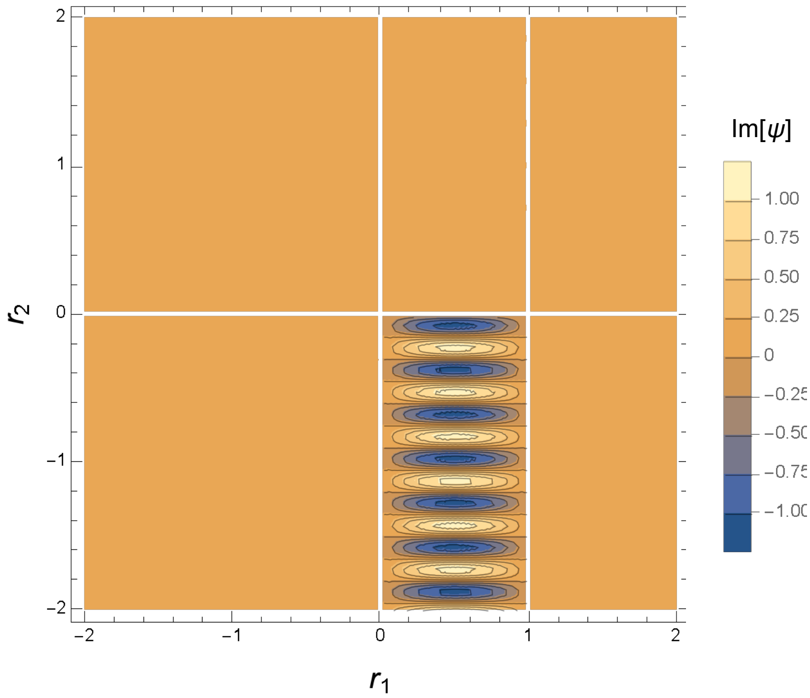
In[69]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[69]=



In[74]:=
```
ContourPlot[Im[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Im[ψ]"]}]
```

Out[74]=

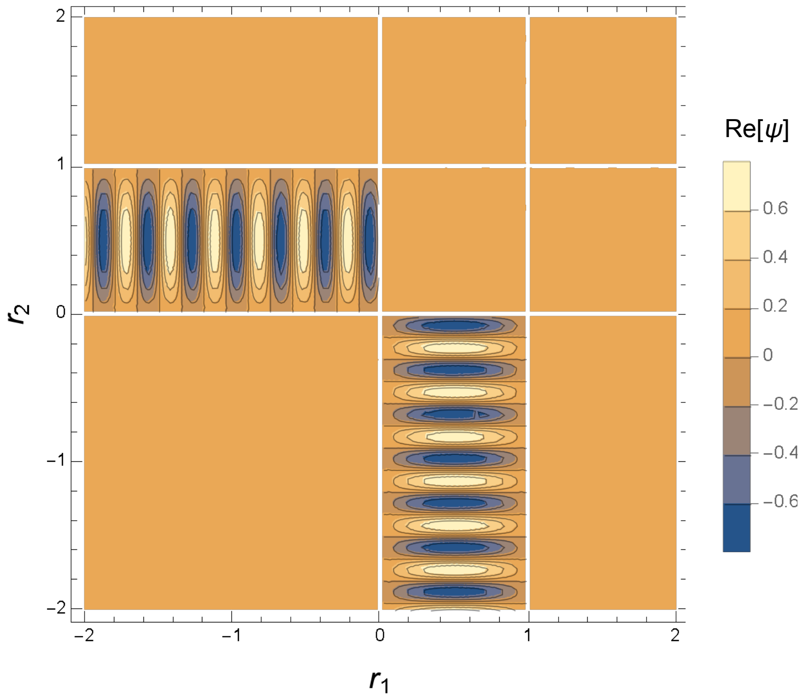


Bosons : For Bosons, we symmetrise the wavefunction

In[77]:=
```
ψ[r1_, r2_] :=
  1/Sqrt[2] (ϕnuc[r1, 1] × ϕscatt[r2, kplot] + ϕnuc[r2, 1] × ϕscatt[r1, kplot]);
```

In[78]:=
```
ContourPlot[Im[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[78]=

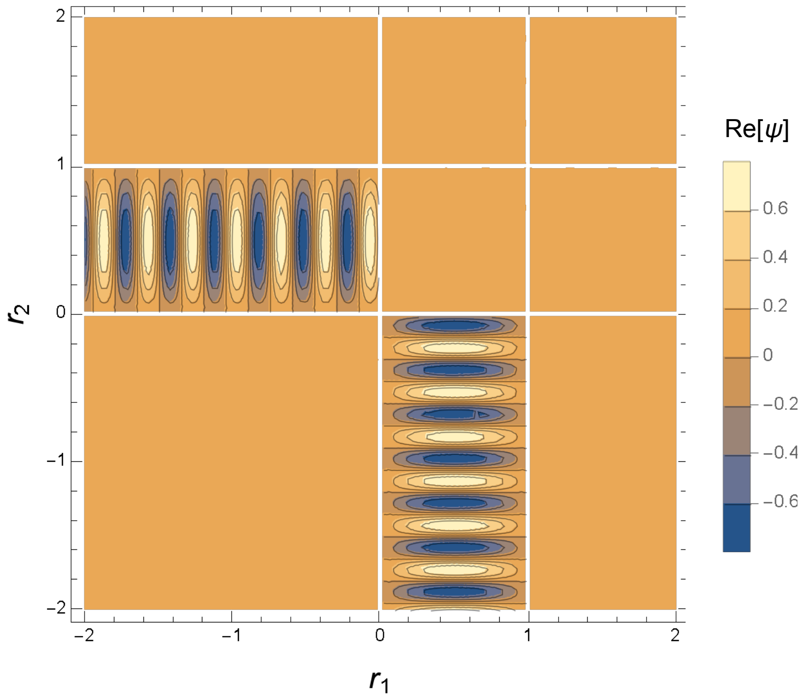


Fermions: For Fermions we anti-symmetrize

In[79]:=
```
ψ[r1_, r2_] :=
  1/Sqrt[2] (ϕnuc[r1, 1] × ϕscatt[r2, kplot] - ϕnuc[r2, 1] × ϕscatt[r1, kplot]);
```

In[81]:=
```
ContourPlot[Im[ψ[r1, r2]], {r1, -Lplot, Lplot},
 {r2, -Lplot, Lplot}, FrameLabel → {Style[r₁, 15], Style[r₂, 15]},
 PlotLegends → {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}]
```

Out[81]=



In this case, since by constructions the particles do not enter the same region, it does not actually matter for any observable which of the three wavefunctions we take.

(part c (i))

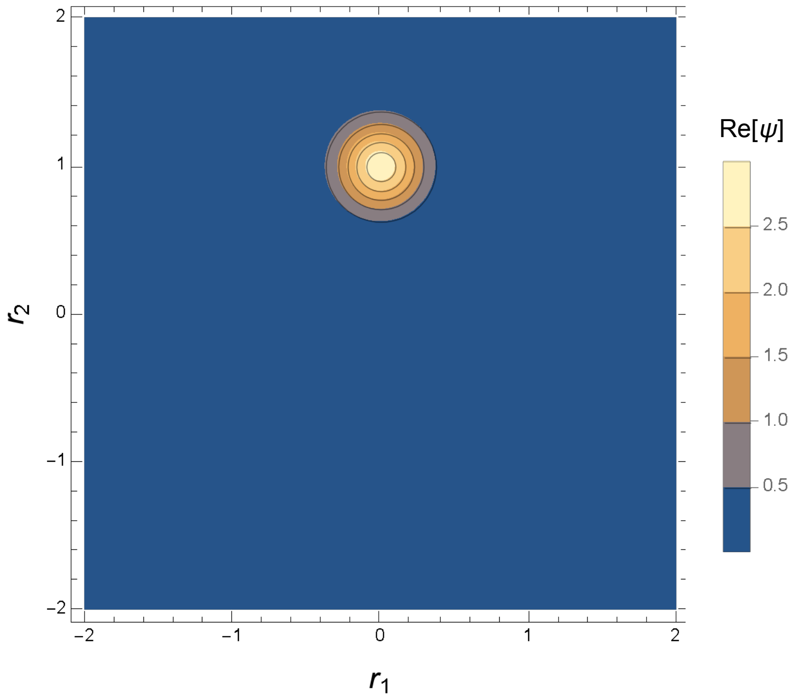
distinguishable: Here we are essentially reproducing the figures in example 5 of the lecture.

In[92]:=
```
ϕ[x_, x0_, σ_] := (1/(π σ^2)^(1/4)) Exp[-(x - x0)^2/2/σ^2];
xA = 0;
xB = 1;
σ = 0.2;
ψ[r1_, r2_] := ϕ[r1, xA, σ] × ϕ[r2, xB, σ];
```

In[99]:=
```
Lplot = 2;
```

In[100]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
 FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
  {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[100]=

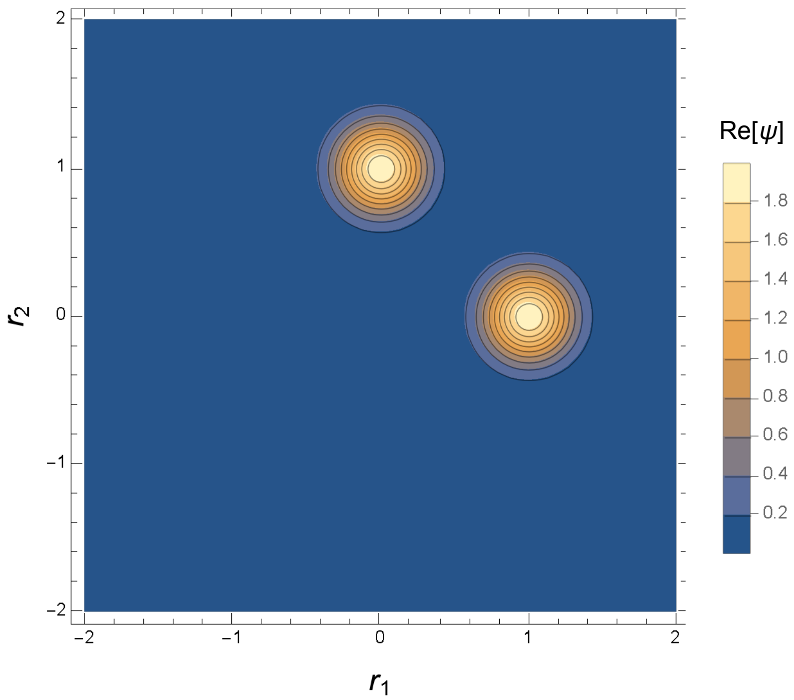


Bosons : For Bosons, we symmetrise the wavefunction

In[101]:=
```
ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] + ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);
```

In[103]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
 FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
  {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[103]=

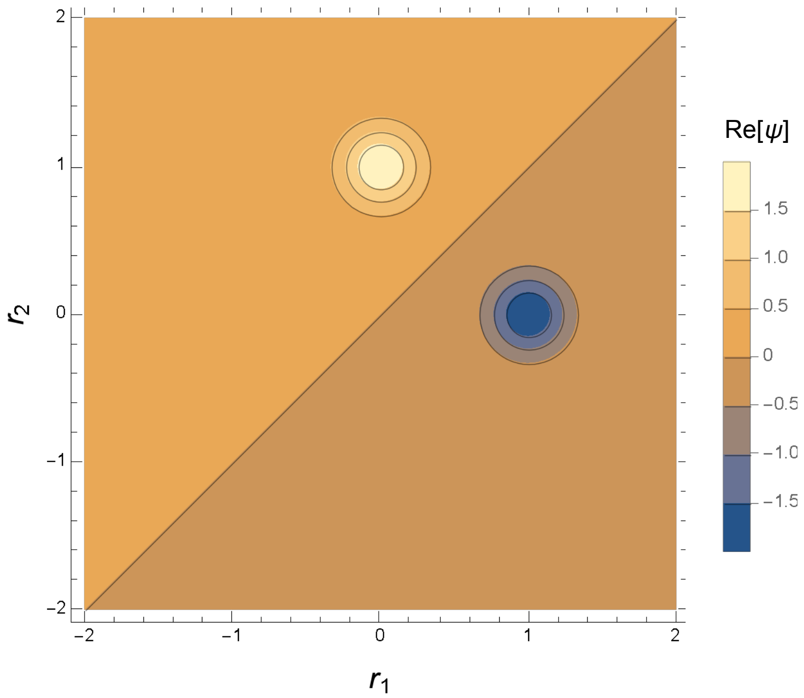


Fermions: For Fermions we anti-symmetrize

```
In[104]:= ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] - ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);

In[107]:= ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
  FrameLabel → {Style[r1, 15], Style[r2, 15]}, PlotLegends →
   {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[107]=



We can see that all three options give different two - body wavefunctions .

(part c (ii))

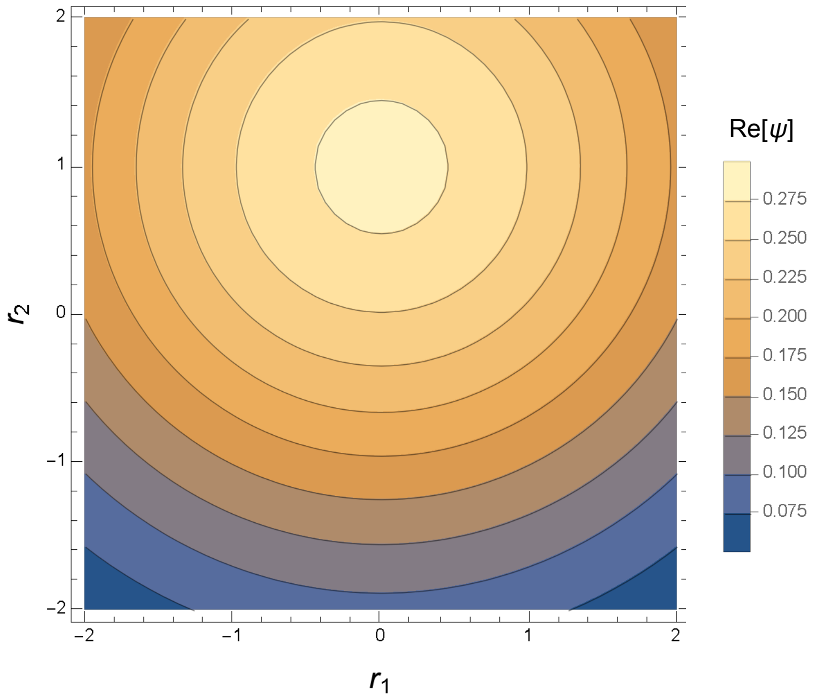
distinguishable: Here we are essentially reproducing the figures in example 5 of the lecture.

```
In[108]:= ϕ[x_, x0_, σ_] := (1/(π σ^2)^(1/4)) Exp[-(x - x0)^2 / 2 / σ^2];
xA = 0;
xB = 1;
σ = 2;
ψ[r1_, r2_] := ϕ[r1, xA, σ] × ϕ[r2, xB, σ];

In[113]:= Lplot = 2;
```

In[114]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
 FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
  {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[114]=

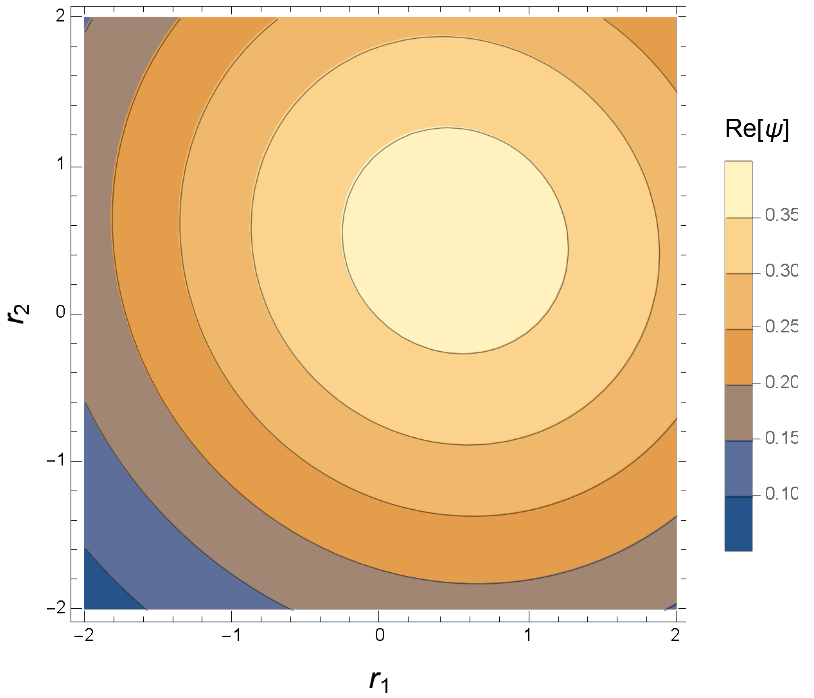


Bosons : For Bosons, we symmetrise the wavefunction

In[116]:=
```
ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] + ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);
```

In[117]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
 FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
  {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[117]=

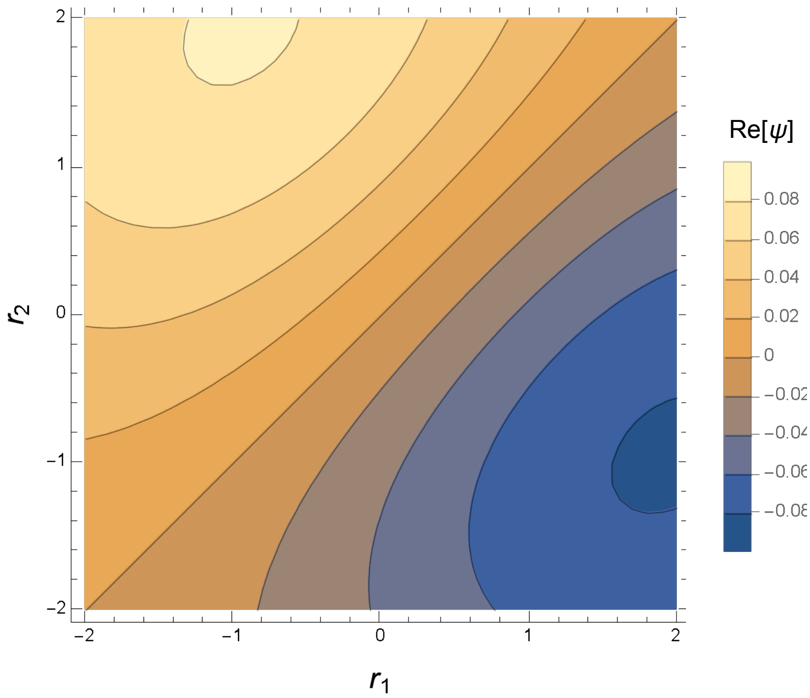


Fermions: For Fermions we anti-symmetrize

In[118]:=
```
ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] - ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);
```

In[119]:=
```
ContourPlot[Re[ψ[r1, r2]], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
 FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
  {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[119]=

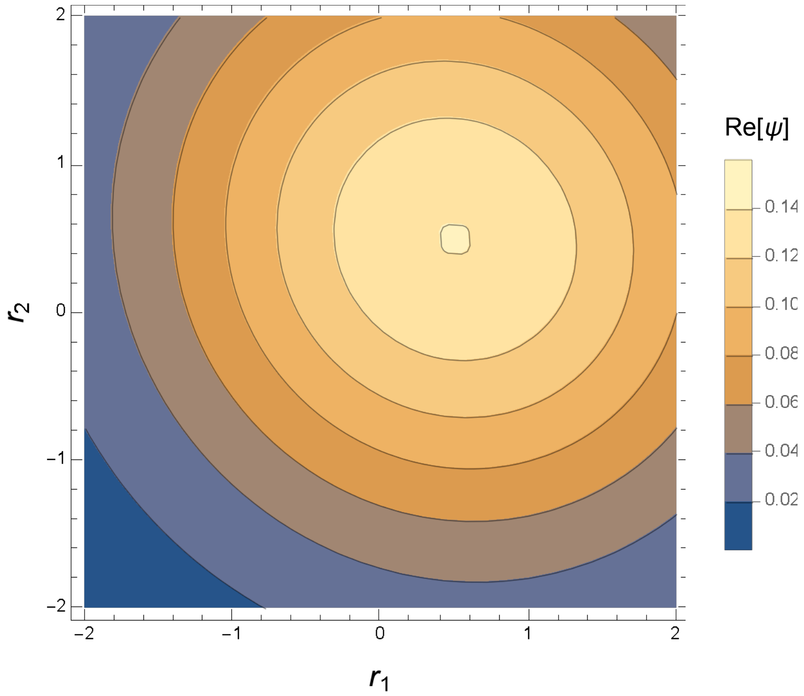


In particular the last two have VERY different probability densities:

In[121]:=
```
ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] + ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);
```

```
In[122]:= ContourPlot[Abs[ψ[r1, r2]^2], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
  FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
   {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[122]=




```
In[123]:= ψ[r1_, r2_] := 1/Sqrt[2] (ϕ[r1, xA, σ] × ϕ[r2, xB, σ] - ϕ[r2, xA, σ] × ϕ[r1, xB, σ]);
```

```
In[124]:= ContourPlot[Abs[ψ[r1, r2]^2], {r1, -Lplot, Lplot}, {r2, -Lplot, Lplot},
  FrameLabel → {Style[r₁, 15], Style[r₂, 15]}, PlotLegends →
   {BarLegend[Automatic, Automatic, LegendLabel → "Re[ψ]"]}, PlotRange → All]
```

Out[124]=