Phys635, MBQM I-Semester 2019/20, Tutorial 4 solution, Fri 8.11.

The objective of this tutorial was to get you to discuss, so there is no "solution". However the thoughts I would have for those prompts for discussion are listed below.

Stage 1 (Questions about the material so far.)

Stage 2 Quantum degeneray: Discuss:

(i) What is conceptually harder about degenerate Fermions than about Bosons?
Due to the Pauli exclusion principle, any interesting Fermionic many-body problem necessarily involves multiple single-particles states

(ii) What is conceptually harder about degenerate Bosons than about Fermions?
Due to the absence of a Pauli exclusion principle, any interesting Bosonic many-body problem typically involves multiple occupancy of a single-particles states.

(iii) Why do we find the temperature where degeneracy effects begin to matter to be about the same, for Bosons and Fermions?

See interpretation in section 3.2.1: Degeneracy effects appear once the de-Broglie matter wave lengths of particles begin to overlapp. This viewpoint makes no difference between Bosons and Fermions.

(iv) How does statistics affect atomic scattering? Which species has which partial waves?
Fermions and Bosons require an (anti-)symmetric pair wave function. Assuming a single spin species, this means also an (anti-)symmetric spatial relative wave function. For this reason, Bosons have only even angular momenta l in the partial wave expansion, Fermions odd l, which in particular precludes $l = 0$ (no s-wave scattering).

Stage 3 Interacting Fermions:

(i) What is a Fermi liquid?
see discussion in lecture and literature

(ii) What is many-body dressing?
see Eq. (4.40) and image on page 90. In quantum mechanics, we generally call a state $|\,\phi\,\rangle = \sqrt{1-\epsilon}|\,a\,\rangle + \sqrt{\epsilon}|\,b\,\rangle$, to be "a state where $|\,a\,\rangle$ is dressed with $|\,b\,\rangle$". If $|\,\phi\,\rangle$, $|\,a\,\rangle$, $|\,b\,\rangle$ are all many-body states, we refer to this as many-body dressing.

(iii) Why are attractively interacting degenerate Fermions fundamentally different from repulsive ones?

See discussion in lecture: Attractive interactions give rise to pairing (see below) and thus make the simple filled Fermi sea a bad starting point for perturbation theory.

(iv) What is a cooper pair?
It is the bound-state of two opposite spin Fermions in the presence of a Fermi-sea blocking momenta up to k_F*, which can then arise due to arbitrarily weak interactions. In solids, two electrons can pair up by effective attractive interactions mediated by lattice phonons. These can have ranges much larger than the screened Coulomb repulsion.*

(v) What is the BCS state?
The complex many-body state that we obtain when translating the cooper pair concept to all-vs-all interactions. Electrons with opposite spins and moments are paired up (or not) all the way up the Fermi surface. See lecture.

(vi) Why/when are degenerate Fermi gases superfluid?
The spectrum has a gap for non-zero pairing field Δ*, so in that case they become superfluid.*