Phys635, MBQM I-Semester 2019/20, Tutorial 3 solution, Thu 17.10.

The objective of this tutorial was to get you to discuss, so there is no "solution". However the thoughts I would have for those prompts for discussion are listed below.

Stage 1 (Questions about the material so far.)

Stage 2 Mean-field theory: Discuss:

(i) What is "mean-field" about mean-field theory?
A single particle would experience an interaction that depends on the positions of all the other particles in $|\psi(\mathbf{r}_1, \cdot\mathbf{r}_N)|^2$*. If more of the others are close, interaction energy is higher else lower. Now the positions of other particles in turn might depend on each other, giving rise to* <u>*correlations*</u>*. These are neglected in MFT, where the interaction is just calculated using the mean (probability) to find a particle in a certain place (e.g. at the point of the first atom). In GP theory, the Ansatz for* ψ *then also has no correlations, but the actual assumption in MFT is just that those are small/unimportant.*

(ii) What are the most important assumptions about the atomic gas [for MFT to be valid]?
Very low temperature, such that condensation happens and s-wave scattering is valid. Very dilute, such that interaction range $\ll$ *interatomic spacing. Weak interactions.*

(iii) What is the interpretation of the condensate wavefunction ?
It is the wavefunction ϕ *shared by all the atoms such that* $\psi(\mathbf{r}_1, \cdot\mathbf{r}_N) = \prod_k \phi(\mathbf{r}_k)$*. This massively simplifies the many-body problem, since we only need to know about a 3D wave function, rather than a* $3 \times N$ *D one. Depending on taste, the condensate wave function maybe normalised to N instead of* 1*.*

(iv) If you want to theoretically study a certain Bose-Einstein condensate experiment (after condensation has happened), which equations can you use? What do they tell you? What input information do you need?
We can use the TIGPE and TDGPE. The first one gives us the shape of the atomic density and momentum distributions. Its input information is the external potential felt by the atoms, their scattering length, mass and the number of atoms. The TDGPE then can tell us the response of the experiment to time-dependent changes such as changing the potential, or inserting new ones, or changing the interactions with a Feshbach resonance (see week 12).

Stage 3 Bogoliubov-excitations:

(i) What are "Bogoliubov-excitations"?
They are the quasi-particles of a BE condensed systems, which means that

we can write the Hamiltonian in a diagonal form with only quasi-particle number operators, assuming that the density of quasi particles is low. If the Hamiltonian takes that form, that means that quasi-particles no-longer interact (which means that we have taken into account the most important effects of interactions correctly). Physically, in BEC, Bogoliubov-excitations are sound waves at large wavelengths (compared to the healing length) and just the excitation (momentum kick) of a single individual atom at small wave-length (or large energies/momenta)

(ii) What can we learn from them?
Their properties depend on the density and interaction strength, so we may learn that from e.g. finding a phonon-spectrum. BdG energies also govern whether a certain setup is stable.

(iii) Under which conditions can you learn about their time-evolution from the GPE?
If they are "macroscopically occupied". The same mode-shapes and energies arise when asking about the effect of a small perturbation of the condensate mean-field.

(iv) How would you create any?
Any sudden change of the Hamiltonian will create some, to be more controlled we want a small change only.