Phys635, MBQM I-Semester 2019/20, Tutorial 2 solution, Wed 18.9.

The objective of this tutorial was to get you to discuss, so there is no "solution". However the thoughts I would have for those prompts for discussion are listed below.

Stage 1 (Questions about the material so far.)

Stage 2 Use a math plotting tool (such as mathematica) to explore Eq. (3.12). If you copy paste the following lines into mathematica, you can start comparing BE distributions $m(E_b)$ for two different sets of parameters.

kbT1 := 2;
kbT2 := 1;
mu1 := -2;
mu2 := -2;
Plot[{1/(Exp[(Eb -mu1)/kbT1] - 1), 1/(Exp[(Eb -mu2)/kbT2] - 1)}, {Eb, 0, 5}]

(i) Let us assume a constant density of states $g(E)$ for simplicity. Confirm that when you reduce the temperature, the total number of particles $N = \int_0^\infty dE g(E) m(E_b)$ does down.
Solution: Let $T_1 > T_2$, you see that $m(E_b, T_1) > m(E_b, T_2)$ for all values of E_b, hence you know the statement is correct without doing any integration.

(ii) Suppose you want to keep the total number constant [while reducing the temperature] what do you do?
Since the only other control knob in the distribution function is μ, we have to adjust that one. It is negative, we have to increase it towards 0.

(iii) We cannot have $\mu > 0$. Can you find a way to keep the total number constant once $\mu = 0$ and you further reduce the temperature?
Not as before. As discussed in the lecture, the only way out is assuming the ground-state is macroscopically occupied (hence BEC).

Stage 3 Quantum Fields: Discuss the following in your team, then on your table. Use the board as well.

(i) Suppose you want to solve Eq. (2.33) for the quantum field operator in some brute force manner. How could you try this in principle? When does it work? When does it not work?
Solution: We could give a restricted range of M elements of some some single particle basis, e.g. oscillator states $\{\varphi_0(\mathbf{x}), \cdots, \varphi_M(\mathbf{x})$, and then devise some restricted Fock space on that, say $|\, N_0, \cdots, N_M\,\rangle$, with $N_i < N_{max}$. The total number of states scales like N^M, thus we can afford only very small N, M. If the physics of interest can be described in the limit of those small N, M, this approach is fine. Often, however, it cannot and we would

require e.g. $N = 10000$*,* $M = 500$*. Then clearly this does not work (see week 13 also)..*

(ii) What is a quantum field?
It is a "field"(i.e. some function of time and space), which is made of "quanta"(i.e. photons). As a consequence of this, it does not need to have a well defined value but can undergoes quantum fluctuations. Mathematically, the two features are manifest by combining single-particle modes (Which depend on time and space), with creation and destruction operators for discrete particles (field quanta).

(iii) Which disciplines use quantum fields? How are they used there.
The two most extensive takers, are presumably particle physics and condensed matter physics. In particle physics there are lots of fundamental reasons for their use, such as the ability to write down theories possessing interesting or relevant symmetries. In the core phenomenon of particle physics, scattering of two particles, there is no initial problem with many-particles, however we get one since energies are so high that new particles can be created and destroyed during the collision. In condensed matter physics this typically does not happen for the fundamental particles, but might for quasi-particles (see week 7,11). But in CM, there are typically truly gigantic numbers of particles involved.