

# Phys635, MBQM I-Semester 2019/20, Tutorial 1 solution, Tue 20.8.

*The objective of this tutorial was to get you to discuss, so there is no “solution”. However the thoughts I would have for those prompts for discussion are listed below.*

**Stage 1** (Questions about the material so far.)

**Stage 2** Why is quantum-many-body theory harder than classical many-body theory?

- (i) What information is needed to specify a classical state of  $N$  particles? A quantum state? *Solution: For the classical state, we write down e.g. a phase space point  $[\mathbf{r}_1, \dots, \mathbf{r}_N; \mathbf{p}_1, \dots, \mathbf{p}_N]$ . In quantum theory a many-body wave-function  $\psi(\mathbf{r}_1, \dots, \mathbf{r}_N) \in \mathbb{C}$ .*
- (ii) Invent a way to “quantify” the volume of that information? How does either scale as the number of particles gets larger? *Solution: Suppose we limit the number of available positions/momenta to  $M$  (or in QM the number of modes to  $M$ ). Then the classical phase-space vector contains  $2N$  real numbers. The QM wavefunction (Eq. (1.24)), contains  $M^N$  complex numbers.*
- (iii) In terms of the classification of many-body states seen in the lecture, which aspect is “causing the trouble”? *Solution: Entanglement. If it wasn’t for entanglement (i.e. we look at a separable state), the information contained again reduces to  $M \times N$  complex numbers (why?), which is not so much worse than classical (we say it has the same scaling with  $N$ ).*

**Stage 3** Let’s chat about what to expect later in lecture. Consider the attached two diagrams for Bosons or Fermions in a harmonic trap. The temperature where it becomes important whether two particles can enter the same state or not, is called “degeneracy temperature”.

- (i) Why would it be not important for some temperatures? *Solution: At large temperatures each particle has a high probability  $p$  to be in a fairly large energy state,  $p \sim \exp(-E/[k_B T])$ . Since this means that a large number of states are available for each particle, the chance of them “wanting to be” in the same single particle state is fairly small. In that case it also obviously doesn’t matter whether they are allowed to be in that same state.*
- (ii) Make a rough estimate of the degeneracy temperature. *Solution: States with  $E \sim k_B T$  become likely. Thus we want  $k_B T \gg N \hbar \omega$  in order for having a larger number of available states than particles.*
- (iii) Suppose we are slowly reducing the temperature of the system. Discuss what might happen in the context of section 2.2.2 of the lecture. *Solution: At temperatures below the transition temperatures, if the system can equilibrate, more than one particle will “want to” get into the same state. For Fermions this will be forbidden due to Fermi blocking. For Bosons it*

will actually speed up, so once there are a few seed Boson in the ground-state, Bose-enhancement will accelerate the actual process of condensation. Importantly, like in 2.2.2., for having any transitions between harmonic oscillator states at all, we need additional terms in the Hamiltonian beyond the harmonic oscillator, for example interactions.

**Stage 4** Think about the numerical component of assignments. In the first assignment, we have seen an exemplary numerical solution of the TDSE for two coupled harmonic oscillators.

- (i) What is conceptually different when you are solving this on a computer, compared to an analytical solution? *Solution: Analytically we have all functions in a continuous space of  $\mathbf{x}$  and  $t$ . In a computer we cannot store a function at an infinite number of coordinates. So we have to discretise both time and space into what is called a lattice  $[t_0, t_1, t_2, t_3, \dots]$   $[\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \dots]$ .*
- (ii) How do you deal with the Laplacian? *Solution: We can calculate it using a Fourier transform, using  $f(x) = \int_{dk} e^{ikx} \tilde{f}(k)$  and thus  $\frac{\partial^2}{\partial x^2} f(x) = \int_{dk} (-k^2) e^{ikx} \tilde{f}(k)$ .*
- (iii) How do you deal with the  $\frac{\partial}{\partial t}$ ? *Solution: Very basic algorithm (Newton's method) would be:  $\frac{\partial}{\partial t} \psi(t) \rightarrow (\psi(t + \Delta t) - \psi(t)) / \Delta t$  for some discrete time step  $\Delta t$ , then solve TDSE for  $\psi(t + \Delta t)$ .*
- (iv) What can go wrong with either? *Solution: If we choose an insufficient number of discrete points in space or time, the solution we get will be a very bad approximation of the continuous function  $\psi(\mathbf{x}, t)$  that we would like to get.*