PHY635, I-Semester 2019/20, last Assignment 6

Instructor: Sebastian Wüster
Due-date: TA-Class, 15.11.2019

Student Name: **ID #:**
Student Name: **ID #:**
Student Name: **ID #:**
Student Name: **ID #:**

(1) Degenerate Fermi gases
(a) Following the procedure of the lecture, calculate the energy of hole or particle excitations in a Fermi liquid (homogeneous system as in lecture) to first order perturbation theory in the interaction between different spin Fermion. Then take their energy relative to the unperturbed Fermi sea. Make a graph of particle/hole energy as a function of k and discuss your results. [*5 points*]

(b) Consider a repulsively interacting balanced spin mixture of Fermions as in chapter 4.9. Assuming the many-body quantum state is given by a thermal density matrix at temperature T for *quasi-particles* (represented by the operators in (4.63), (4.65)), show that the (constant) Fock field vanishes, $F = U_0\langle\hat{\Psi}^\dagger_\uparrow(\mathbf{x})\hat{\Psi}_\downarrow(\mathbf{x})\rangle = 0$. We had used this in the actual derivation of the quasi-particles, so you have now shown that also this was self-consistent. [*Hint: Start as we did when deriving the gap-equation. Also re-read section 3.4.4. for some similar manipulations, there for Bosons and trap-states instead of momentum states*]. [*5 points*]

(2) BCS State and Cooper pairs. Explore the relation ship between the BCS many-body state and cooper pairs. [*Some of the answers to the following questions can be found in the lecture notes. This question is supposed to encourage you to go through that I detail. I suggest you do it from scratch without looking at the notes, and only use them if you get stuck or to confirm your results. For you hand-in, it is then of course even more important that you write a lot of text to show that you understood everything. Copying the lecture notes will not be sufficient*]

(a) In the lecture we defined the Cooper pair creation operator:

$$\hat{c}^\dagger = \int d^3\mathbf{x} \int d^3\mathbf{y}\, \psi_0(\mathbf{x},\mathbf{y})\hat{\Psi}^\dagger_\uparrow(\mathbf{x})\hat{\Psi}^\dagger_\downarrow(\mathbf{y}), \tag{1}$$

Write explicitly the state $\hat{c}^\dagger|\,0\,\rangle$, where $|\,0\,\rangle$ is the vaccuum state, in first quantized representation. [*3 points*]

(b) Express $\hat{c}^\dagger$ in terms of momentum creation operators, instead of position space field operators. [*3 points*]

(c) Show that the coherent state of Cooper pairs $|\,\gamma\,\rangle = \mathcal{N}\exp\left[\gamma\hat{c}^{\dagger}\right]|\,0\,\rangle$ is the same as $|\,\psi BCS\,\rangle = \prod_{\mathbf{k}}\left(u_{\mathbf{k}} + v_{\mathbf{k}}\hat{a}^{\dagger}_{\uparrow\mathbf{k}}\hat{a}^{\dagger}_{\downarrow\mathbf{k}}\right)|\,0\,\rangle$. [*4 points*]

(3) Superconductivity and Superfluidity Write a short (about one page) essay about superconductivity and Fermion superfluidity based on what you learnt in this lecture and/or read elsewhere. Use google also. Focus on the essentials only. Make sure to touch the points below. [*10 points*][*Make sure this is your own text. Plagiarism will strictly not be tolerated*]

- What is a cooper pair? What are the essential requirements to have one?
- What is the superconducting state? State of superfluid fermions?
- What phenomena are characteristic for superconductors or superfluids?
- Why are superconductors superconducting?
- What are open questions or future goals in this area?

(4) Bosonic versus Fermionic 2D Scattering
The template file `Assignment6_phy635_program_draft_v2.xmds` is set up to solve the Schrödinger equation for the relative coordinate $\mathbf{r} = [r_x, r_y]$ for a pair of distinguishable particles of mass m undergoing a scattering process in 2D, interacting with a Gaussian potential $U(\mathbf{r}) = A\exp\left(-\mathbf{r}^2/2/\sigma_{\text{int}}^2\right)$. We use dimensionless units where $\hbar = 1$. The particles are initialised in a wavepacket $\psi(\mathbf{r}) = \mathcal{N}\exp(-(\mathbf{r}-\mathbf{r}_0)^2/(2\sigma^2))\exp\left[i\mathbf{k}_{\text{ini}}\mathbf{r}\right]$, such that they collide (reach $\mathbf{r} = 0$) at half the simulation time. [*Note:* $\sigma \neq \sigma_{int}$].
(4a) The range of the interaction potential set up in the code initially is $\sigma_{\text{int}} = 0.002$. Calculate the impact parameter $d_{l=1}$ for which the particles, considered classically, would have angular momentum corresponding to the quantum mechanical $l = 1$ ($|L| = \hbar\sqrt{l(l+1)}$) and compare with $\sigma_{\text{int}} = 0.002$. [*2 points*].

(4b) Run the code and analyse the resultant scattered wave with the attached script `slideshow_ultracold_scattering_v2.m`. This is set up to amplify small intensity features in the matter wave, since scattering is kept weak (small A), so that perturbation theory remains valid. Discuss your findings, relate them to the result of (4a). [*4 points*]

(4c) Now modify the code such that it considers the scattering of (i) indistinguishable Bosons, (ii) indistinguishable Fermions. Re-run the scattering of (b) and discuss and interpret your results. [*4 points*]

(4d, bonus) Explore what happens when you now significantly extend the range of the potential σ_{int}.