PHY635, I-Semester 2019/20, Assignment 4

Instructor: Sebastian Wüster
Due-date: TA-Class, 18.10.2019

Student Name: **ID #:**
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(1) Gross-Pitaevskii equation Consider some special solutions of the TIGPE (3.44). Let us assume a 1D system without any external trap $V(x) = 0$.

(i) For the case of attractive interactions, $U_0 < 0$, show that one solution for N atoms at a chemical potential μ is given by

$$\phi_0(x) = A\,\mathrm{sech}(x/\xi), \qquad (1)$$

This is called a bright soliton. While showing it is a solution, find the values for A and ξ required. Feel free to use Mathematica as much as desired, but describe the steps for arriving at the solution manually. [*5 points*]

(ii) Plot the atom density for some realistic parameters. Start with the mass of Litihum and $a_s = -1a_0$ (where a_0 is the Bohr radius) but then use $U_0 = U_{1d} = \frac{4\pi\hbar^2 a_s}{m}/(2\pi\sigma_\perp^2)$ with $\sigma_\perp = (2\pi)10000$ Hz. We have to use U_{1d} because of our one dimensional calculations. Also use $N = 5000$. Then plot it for some varied values of N and $a_s < 0$. [*5 points*]

(iii) Google the term soliton and explain in your own words what it implies. [*5 points*]

(2) Bogoliubov excitations and stability Consider the Bogoliubov dispersion relation given in the lecture.

(i) Suppose we have a homogenous condensate with attractive interactions $U_0 < 0$. Discuss the BdG energies, depending on the excitation wave-number q. How does the time-evolution of the modes look, according to Eq. (3.73) of the lecture. Identify stable and unstable modes. [*5 points*].

(ii) Suppose you have a finite size system, such as a box of size L. How would this change the stability picture? [*5 points*]

(iii) Explain in your own words why excitations with wavelengths larger than the healinglength are referred to as “collective excitations” (or sound waves) and those with shorter wavelengths as “single particle excitations”. [*5 points*]

(3) Numerical Solution of Gross-Pitaevskii equation

The template file `Assignment4_phy635_program_draft_v1.xmds` is set up to first find a ground-state of the TIGPE using"imaginary time evolution", and then evolve that state in time. The potential imposes a hard-walled box, as discussed in section 3.3.5 of the lecture. During the ground-state finding the codes uses an interaction strength U_{ini} the time evolution uses U_{fin}.

(4a) Verify the ground-state looks like the figure in section 3.3.5. Checkout the size of the edge transition region for various values of U_{ini} to confirm Eq. (3.50). Also check that the found ground-state does note evolve in time. [*5 points*]

(4b) Now change the interaction strength U_{fin} used during time-evolution to an attractive value (< 0). Describe what you see and make some plots. Discuss wrt. your results of question 2. [*5 points*]