Consider a two mode problem, with single particle states $|\varphi_1\rangle$, $|\varphi_2\rangle$, and associated operators $\hat{a}_1^\dagger|0\rangle = |\varphi_1\rangle$ etc. The second quantized Hamiltonian is

$$\hat{H} = E_1\hat{a}_1^\dagger\hat{a}_1 + E_2\hat{a}_2^\dagger\hat{a}_2 + J(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1)$$

For that Hamiltonian, change basis to states $|\varphi_\pm\rangle = \frac{1}{\sqrt{2}}(|\varphi_1\rangle \pm |\varphi_2\rangle)$, with associated operators $\hat{a}_\pm^\dagger|0\rangle = |\varphi_\pm\rangle$ etc. < br> You shall find:

- $\hat{H} = \frac{E_1+E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-\right) + \frac{E_1-E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_- + \hat{a}_-^\dagger\hat{a}_+\right) + J(\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-)$
- $\hat{H} = \frac{E_1+E_2}{2}\hat{a}_+^\dagger\hat{a}_+ + \frac{E_1-E_2}{2}\hat{a}_-^\dagger\hat{a}_- + J(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-)$
- $\hat{H} = \frac{E_1+E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-\right) + \frac{E_1-E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_- + \hat{a}_-^\dagger\hat{a}_+\right) + J(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-)$
- $\hat{H} = \frac{E_1+E_2}{2}\hat{a}_+^\dagger\hat{a}_+ + \frac{E_1-E_2}{2}\hat{a}_-^\dagger\hat{a}_- + J(\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-)$

Correct Answer

Score: 1

Correct Answer: $\hat{H} = \frac{E_1+E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-\right) + \frac{E_1-E_2}{2}\left(\hat{a}_+^\dagger\hat{a}_- + \hat{a}_-^\dagger\hat{a}_+\right) + J(\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-)$

The following first-quantized Hamiltonian describes electrons in a magnetic field, with some ficticious interactions among them. All terms in it are labelled (A,B,C...). Please segregate them into classes of N-Body Operators, that means for each label, identify the N and insert it into the blank.

$$\hat{H} = \sum_n \Bigg[\underbrace{\frac{1}{2m}(\mathbf{p}_n + e\mathbf{A})^2}_{=A} - \underbrace{\frac{Ze^2}{4\pi\epsilon_o r_n}}_{=B} + \underbrace{\frac{g_s\mu_B}{\hbar}\mathbf{B}\cdot\mathbf{S}_n}_{=C} + \underbrace{\zeta(r_n)\mathbf{L}_n\cdot\mathbf{S}_n}_{=D} + \sum_m \Big\{ \underbrace{\chi(r_n)\mathbf{r_n}\times\mathbf{p_m}}_{=E} + \sum_k \underbrace{\xi(r_m)\mathbf{r_k}\cdot\mathbf{p_n}}_{=F} \Big\} \Bigg]$$

Hamiltonian

Variables are: e electron charge, m mass, Z nuclear charge number, ϵ_0 vacuum permeability, μ_B Bohr magneton, g_s dyromagnetic ratio, ζ, χ, ξ some function of their arguments.

Term classification:

A: 1

B: 1

C: 1

D: 1

E: 2

F: 3

All blanks should be integers.

Essentially, you had to count the number of different summation indices in each term, since those label the particles. One index n = single body term, two indices nm = two-body term, three indices m,p,n = three body term

The following second-quantized Hamiltonian describes particles in a Harmonic trap, with single particle eigenstates $\hat{H}_0|\varphi_n\rangle = E_n|\varphi_n\rangle$ and associated creation and destruction operators $\hat{a}_n$ etc. There are some ficticious interactions among them. All terms in it are labelled (A,B,C...). Please segregate them into classes of N-Body Operators, that means for each label, identify the N and insert it into the blank.

$$\hat{H} = \sum_n \Bigg[\underbrace{\hbar\omega \hat{a}_n^\dagger \hat{a}_n}_{=A} + \sum_m \Bigg(\underbrace{T \hat{a}_n^\dagger \hat{a}_m}_{=B} + \sum_{k,l} \Bigg\{ \underbrace{F \hat{a}_n^\dagger \hat{a}_m^\dagger \hat{a}_k \hat{a}_l}_{=C} + \sum_{n',m',k',l'} \Bigg[\underbrace{U \hat{a}_n^\dagger \hat{a}_m^\dagger \hat{a}_k^\dagger \hat{a}_l^\dagger \hat{a}_n \hat{a}_m \hat{a}_k \hat{a}_l}_{=D} \Bigg] \Bigg\} \Bigg) \Bigg]$$

Hamiltonian

Term classification:

A: 1

B: 1

C: 2

D: 4

All blanks should be integers.

One and two body terms as in lecture. Essentially count the total number of a,a^\dagger operators and divide by two.

Consider Bose atoms that can be in two different electronic (internal) states $|g\rangle$ and $|r\rangle$, created (destroyed) by $\hat{a}_g^\dagger e$, $\hat{a}_r^\dagger e$ ($\hat{a}_g e$, $\hat{a}_r e$). Let the Hamiltonian be

$$\hat{H} = \frac{\Omega}{2}(\hat{a}_g^\dagger \hat{a}_r + \hat{a}_r^\dagger \hat{a}_g) + \frac{\Delta}{2}\hat{a}_r^\dagger \hat{a}_r(\hat{a}_r^\dagger \hat{a}_r - 1),$$

with two parameters Ω and Δ. In the space of the three Fock-states $|N_g N_r\rangle \in \{|3,0\rangle, |2,1\rangle, |1,2\rangle, |0,3\rangle\}$, write the matrix-form of $\hat{H}$.
The matrix form is

$$\begin{bmatrix} F & \sqrt{3}A/2 & D & D \\ \sqrt{3}B/2 & F & A & D \\ E & B & C & \sqrt{3}A/2 \\ E & E & \sqrt{3}B/2 & 3C \end{bmatrix},$$

with
A: 1
B: 1
C: 2
D: 0
E: 0
F: 0
(insert only the following code numbers: 0=zero, 1=Omega, 2=Delta, 3=two Delta, 4=two Omega)

Unfortunately, partial credit for this question is not supported, thus carefully check ALL blanks before submitting.

A system of identical non-interacting Bosons is at chemical potential $\mu = -5$ and temperature $k_B T = 3$ (we have used some arbitrary units). Find the mean number of Bosons in the three single particle states $|\varphi_{A,B,C}\rangle$ with energies $E_1 = 0.50$, $E_2 = 1$, $E_3 = 5$.

$\bar{m}_1 =$ 0.19

$\bar{m}_2 =$ 0.16

$\bar{m}_3 =$ 0.03 0.038

two significant digits = the first two non-zero ones

The blanks accept decimal numbers. Please round to two significant digits only. Examineer will accept numbers if they are close enough. If you brought no calculator, use the one included in examineer.

The critical temperature for a harmonically trapped Bose-Gas of 10000 Rubidium atoms is 50 nK. You find that 8750 of these atoms are condensed. What can you say about the temperature (in nK) of the gas?

Answer: Its temperature is 25 nK.

Round to the nearest integer, small deviations will be allowed.

Correct Answer

Score: 1

Correct Answer: Answers in order of appearance of blanks:
25

inverting the formula from lecture: $T = (1-N_c/N)^{1/3} T_C$