Fill in the blank and click lock.

For the pure quantum state $|\mathrm{Psi}\rangle = N\left(3 \mid \phi_a\rangle + 2 \mid \phi_b\rangle\right)$, where N is a normalisation factor, the corresponding density matrix is

$$\hat{\rho} = N^2 \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

Here the elements are

$\rho_{11} =$ 9,

$\rho_{12} =$ 6,

$\rho_{21} =$ 6,

$\rho_{22} =$ 4.

Write the elements ρ_{ij} as shown in the equation, they **do not** *include the normalisation factor N*.

Correct Answer

Score: 1

Correct Answer: Answers in order of appearance of blanks:

9

6

6

4

Suppose we know all eigenstates and eigenenergies of the time independent Hamiltonian

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle.$$

We want to expand some given state as $|\Psi\rangle = \sum_n X_n|\phi_n\rangle$

- $X_n = \langle\Psi|\phi_n\rangle$
- $X_n = |\Psi\rangle$
- $X_n = e^{-iE_nt/\hbar}$
- $X_n = \langle\phi_n|\Psi\rangle$
- $X_n = |\phi_n\rangle$

Correct Answer

Score: 1

Correct Answer: $X_n = \langle\phi_n|\Psi\rangle$

Fill in the blank and click lock.

Suppose we know all eigenstates and eigenenergies of the time independent Hamiltonian $\widehat{H} \mid \phi_n\rangle = E_n \mid \phi_n\rangle$. Let $E_a = 1$, $E_b = 2$, $E_c = 4$, and assume the system is in the state $\mid \Psi\rangle = \frac{\mid \phi_a\rangle}{\sqrt{2}} + \frac{\mid \phi_b\rangle}{2} + \frac{\mid \phi_c\rangle}{2}$. This state is normalized to 1 .

A: Which mean value will we get out of a large number of energy measurements on this state?

2

B: Out of 100 attempts, how often will we measure the energy value that you found in question part-A? (25 * 1)

C: Out of 100 attempts, how often will we measure the energy value 4 ? (25 * 1)

Correct Answer

Score: 1

Correct Answer: Answers in order of appearance of blanks:

2

25

25

Suppose we know all eigenstates and eigenenergies of the time independent Hamiltonian

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle.$$

Let the system at t=0 be in the state $|\Psi(0)\rangle = (|\phi_a\rangle + |\phi_b\rangle)/\sqrt{2}$. What is the state at time t?

- $|\Psi(t)\rangle = e^{-i[E_a+E_b]t/\hbar}(|\phi_a\rangle + |\phi_b\rangle)/\sqrt{2}$
- $|\Psi(t)\rangle = (e^{-iE_a t/\hbar}|\phi_a\rangle + e^{-iE_b t/\hbar}|\phi_b\rangle)/\sqrt{2}$
- $|\Psi(t)\rangle = (|\phi_a\rangle + |\phi_b\rangle)/\sqrt{2}$
- $|\Psi(t)\rangle = (e^{-i[E_a-E_b]t/\hbar}|\phi_a\rangle + |\phi_b\rangle)/\sqrt{2}$

Correct Answer

Score: 1

Correct Answer: $|\Psi(t)\rangle = (e^{-iE_a t/\hbar}|\phi_a\rangle + e^{-iE_b t/\hbar}|\phi_b\rangle)/\sqrt{2}$

Consider the simple harmonic oscillator, with Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2.$$

Defining $\sigma = \sqrt{\hbar/m/\omega}$, which of the following is an eigenstate of $\hat{H}$, up to a normalisation factor?

- $\exp\left[-x^2/2/\sigma^2\right](-12x/\sigma + 8(x/\sigma)^3)$
- $\exp\left[-x^2/2\right](-12x + 4x^2)$
- $\exp\left[-x^2/\sigma^2\right]$
- $\exp\left[ikx\right]$
- $\exp\left[-x^2/2\right](-12x + 8x^3)$

Correct Answer

✖

Score: 1

Correct Answer: $\exp\left[-x^2/2/\sigma^2\right](-12x/\sigma + 8(x/\sigma)^3)$

Fill in the blank and click lock.

Consider the eigenstates of a particle in a 1D box potential: $V(x)$ =infinity, if $x < 0$ or $x > L$, $V(x) = 0$ otherwise. These are given by $\Psi_n(x) = \sqrt{2}\sqrt{\frac{1}{L}}\sin(k_n x)$, with $k_n = \frac{\pi n}{L}$. Let us choose $L = \frac{1}{4}$.

Determine the following expectation values $\langle\Psi_1 \mid U(x) \mid \Psi_1\rangle$ over the additional perturbation $U(x)$ in the state $\Psi_1(x)$.

Hint: Best first sketch all quantities (potential, wavefunction, perturbation).

A: For $U(x) = U_0\,x - \frac{\mathrm{LU}_0}{2}$ with $U_0 = 10$: 0

B: For $U(x) = \delta\left(-\frac{L}{2} + x\right)$, where $\delta(x)$ is the Dirac delta function:

8

C: For $U(x) = 12\,\theta\left(-\frac{L}{2} + x\right)$, where $\theta(x)$ is the Heaviside step function:

6

Correct Answer

Score: 1

Correct Answer: Answers in order of appearance of blanks:

0

8

6