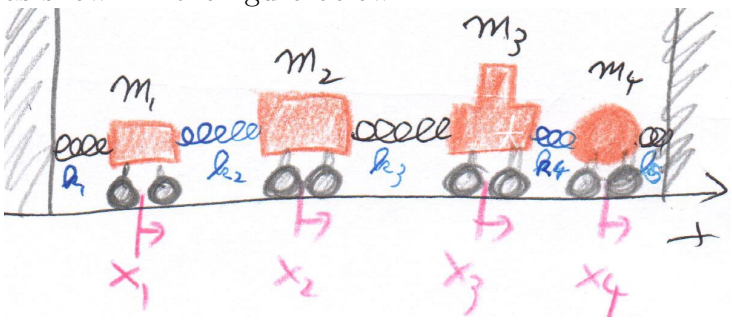
PHY 305, I-Semester 2020/21, Tutorial 7

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss via online (video or audio) conference on a subchannel for your group.

Stage 1 Coupled oscillators Extend the picture of section 3.6.1. to four coupled carts as shown in the figure below:



left: Four coupled carts.

(a) Write down the equations of motion.

(b) Cast those in the form of a matrix equation as done in the lecture.

(c) Without doing the calculation, discuss in your team how you would solve the problem of finding the time-evolution of those four carts from a known initial condition.

Stage 2 Hamiltonian mechanics

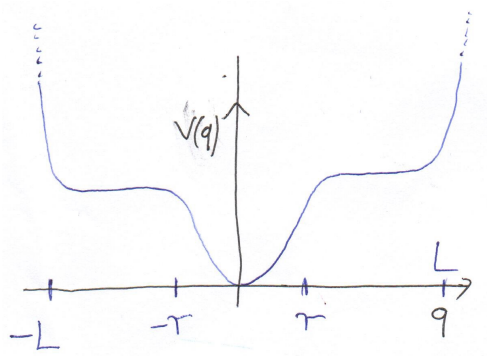
(a) What is the motivation to develop Hamiltonian mechanics? What is phase-space? Why is phase-space a helpful concept? What does Lioville's theorem say? Why is it useful?

(b) To see better why we need a Legendre transformation to go from a Lagrangian to a Hamiltonian, let's go back to the mathematical Legendre transformation on the level of a function, Eq. (4.3). For simplicity, we omit the second argument y. Consider two functions $f(x)$, for which we want to change variables to $u = \partial f/\partial x$: $f_1(x) = ax^2$ and $f_2(x) = a(x+b)^2$. For both of these, (a) find $u(x)$, (b) write a function $\tilde{f}(u)$ by solving your expression for $u(x)$ for x and eliminating x from $f_k(x)$ in favor of u. (c) Instead, find the Legendre transformations $g(u)$ of these two functions. Compare the results of the two approaches, what do you observe?

(c) Now discuss why we need a Legendre transformation to go from a Lagrangian to a Hamiltonian.

Stage 3 Phase space

(a) Consider a particle in a 1D potential $V(q)$ drawn below, with Hamiltonian $H = \frac{p^2}{2m} + V(q)$. PTO



left: Drawing of potential

(b) Based on the drawing and your knowledge of basic physics, draw a qualitative phase-space portrait for that Hamiltonian.