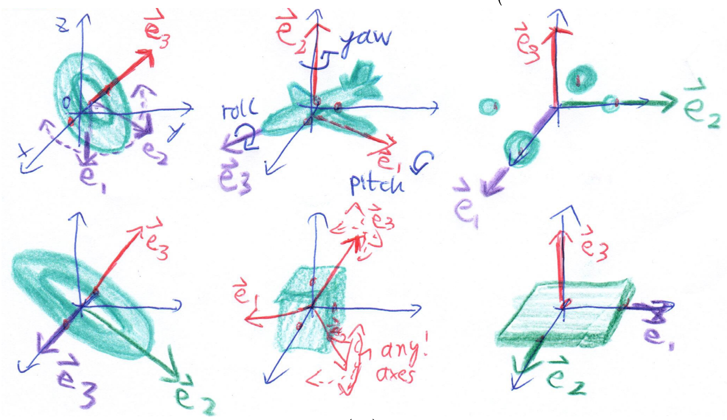
PHY 305, I-Semester 2020/21, Tutorial 6 solution

Stage 1 Moment of Inertia tensors without math:

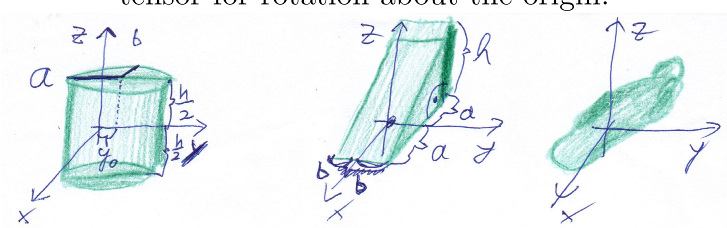
(a) For the following objects (centered on the origin) and rotation axes through the origin, based on what you know from the lecture, find or guess three principal axes and the relative size of the associated moments of inertia. Assume a homogenous solid mass density within the greenish volume. Points where coordinate axes cut through the volume are shown in red for orientation.

Solution: The following figure now contains the principal axes, with colorcoding: (red) largest moment of inertia, (violet) smallest, (green) in between. The way we would guess these, is as follows: If there are obvious symmetry axes or NEAR ssymmetry axes, these must be principal axes. If there are not, think about which axes you guess should give the largest rotational kinetic energy (i.e. for rotation about it, the mass is as far away from the rotation axis as possible). In the end, of course, one should verify the intuition here with a calculation (not in the tutorial).



top: (a) Regular circular torus. (b) Airplane, with the usual pieces and symmetries. (c) 4 Spheres, 2 large, 2 small, all the same distance from the origin. (d) Ellipsoidal torus, with smaller radius along x-axis. (e) Symmetric cube. (f) Cuboid, with withs along axes $w_z < w_x < w_y$.

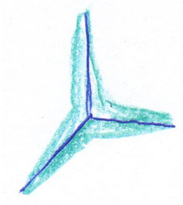
(b) For the following objects, list all obviously zero elements of the inertia tensor for rotation about the origin.



top: (a) Cylinder with ellipsoidal cross section of indicated semi-minor axis b and semi-major axis a, around centre point at $x_0 = 0$ $y_0 > 0$ as shown. (b) Triangular block. (c) Asteroid toutatis (see lecture notes) in random orientation.

Solution: (a) The object as described is symmetric under $z \leftrightarrow -z$ and $x \leftrightarrow -x$ but not under $y \leftrightarrow -y$ (we could reach the latter by a smarter choice of origin). Going to the definition (3.25), then tells us straight away (due to symmetry of the integrand/sum), that $I_{xy} = I_{yx} = 0$, $I_{zy} = I_{yz} = 0$, $I_{xz} = I_{zx} = 0$. So we see that it sufficient for an object to be symmetric along two axes for the moment of inertia tensor to become diagonal. Diagonal elements cannot be zero per definition (that would require vanishing density everywhere), so we do not need to look at those for this question. (b) This object is symmetric under $y \leftrightarrow -y$ but no other axis reflection. Hence $I_{xy} = I_{yx} = 0$, $I_{zy} = I_{yz} = 0$, but we can't say much about I_{xz} and $I_{zx} = 0$ at this point. (c) In a general random direction, there is no reason to expect any matrix element to vanish (unless we accidentally his some orientation where they do).

Stage 2 Discuss what are Euler angles and what are body fixed and space fixed frames. Then assemble an experimental tripod of coordinate axes as for the figure on top of page 73. Maybe out of one corner of a cardboard carton (see figure below), or by attaching three spoons at right angles. Then use that to implement three separate Euler angle rotations and convince yourself that this can get you from any orientation of your axes to any other orientation.



left: Axes tripod out of cardboard carton corner.

(not really a) solution: See lecture 3.2.2. Or also this video . If you watch it, please let me know if it uses the same convention for Euler angles as discussed in the lecture, I did not verify this.

Stage 3 Discuss what is meant by precession and nutation of a heavy top, and why either of them happens. Then try to find or buy a children's top, or some object that can function as one (symmetric in xy, bit bulgy in z, heavy and rigid enough). Try to reproduce precession or nutation as discussed in the lecture. The heavier your top the better, as light ones might be dominated by effects we had not included, such as friction on the table. If your top works well, make a video and share it with others. If your top does not work well, or you could not acquire one, use this online app instead.

Solution: Precession refers to the rotation of the rotation axis around the z-axis. It happens since the torque exerted by gravity on the top always acts sideways (and in a horizontal plane) to the angular momentum. Nutation refers to additional up or down oscillations of the rotation axis, accompanied by slight slowdowns and speedups of the velocity of precession. It happens mathematically since the approximation free Lagrange equations for all three Euler angles of the heavy top couple all three angles nontrivially.