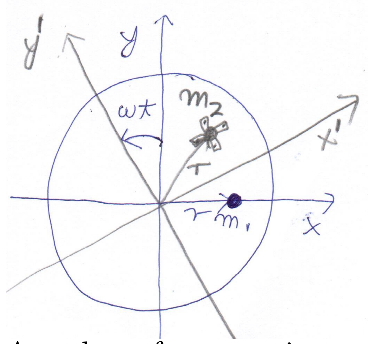
PHY 305, I-Semester 2020/21, Tutorial 5

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss via online (video or audio) conference on a subchannel for your group.

Stage 1 What is the difference between an inertial and a non-inertial frame? What is a fictitious force? Why do we need fictitious forces in non-inertial frames (answer both: mathematically and physically)?

Stage 2 Describe in your own words how the concept of "moment of inertia" and "inertia tensor" arises. What are good and bad choices of coordinates for the calculations of moments of inertia?

Stage 3 (i) *(Rotating frames I)*



left: Consider the blue mass m_1 which slides without friction on the turntable shown, a distance r from the centre of the table. The mass is at rest in the fixed frame x, y. The turntable rotates with angular velocity ω.

Are there forces acting on the mass in the frame that rotates with the turntable? If yes, find them all....

(ii) *(Rotating frames II)* Now consider the second mass m_2 instead, which is initially bolted to the turntable so it is fixed in the frame x', y'. While it is bolted, what are the forces acting on the bolts?

(iii) *(Rotating frames III)* If the mass is released from the bolts, it travels in a straight line in the space fixed frame (x,y). What path does it take in the rotating frame of the turntable (x',y')?

Stage 4 Moments of inertia, cross-products

(i) Find the moment of inertia tensor for the following collections of four masses m_1, m_2, m_3, m_4, for rotations around the $z - axis$. Start with making a drawing of the configuration. Relate this to your understanding of angular momentum and rotational kinetic energy for such a rotation.

- $m_1 = 1$ kg, $\mathbf{r}_1 = [1, 0, 0]^T$ m, $m_2 = 1$ kg, $\mathbf{r}_1 = [-1, 0, 0]^T$ m, $m_3 = 5$ kg, $\mathbf{r}_1 = [0, 0, 0]^T$ m, $m_4 = 5$ kg, $\mathbf{r}_1 = [0, 0, -3]^T$ m.
- $m_1 = 1$ kg, $\mathbf{r}_1 = [2, 0, 0]^T$ m, $m_2 = 1$ kg, $\mathbf{r}_1 = [-2, 0, 0]^T$ m, $m_3 = 5$ kg, $\mathbf{r}_1 = [0, 0, 0]^T$ m, $m_4 = 5$ kg, $\mathbf{r}_1 = [0, 0, -3]^T$ m.
- $m_1 = 1$ kg, $\mathbf{r}_1 = [1, 0, -1]^T$ m, $m_2 = 1$ kg, $\mathbf{r}_1 = [-1, 0, 1]^T$ m, $m_3 = 5$ kg, $\mathbf{r}_1 = [0, 0, 0]^T$ m, $m_4 = 5$ kg, $\mathbf{r}_1 = [0, 0, -3]^T$ m.

- $m_1 = 2$ kg, $\mathbf{r}_1 = [1, 0, -1]^T$ m, $m_2 = 2$ kg, $\mathbf{r}_1 = [-1, 0, 1]^T$ m, $m_3 = 2$ kg, $\mathbf{r}_1 = [1, 0, 1]^T$ m, $m_4 = 2$ kg, $\mathbf{r}_1 = [-1, 0, -1]^T$ m.

(ii) Using the epsilon-tensor and the rule $\sum_i \epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$, show Eq. (2.51) and Eq. (3.23) of the lecture:

$$\begin{aligned} \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}), \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}). \end{aligned} \tag{1}$$

for 3-component vectors **a**, **b**, **c**.