PHY 305, I-Semester 2020/21, Tutorial 4

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss via online (video or audio) conference on a subchannel for your group.

Stage 1 *(Symmetries)*

(i) The following Lagrangian describes one particle in 2 dimensions of space.

$$\mathcal{L} = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) - V_0(q_1^2 + q_2^2)^2. \tag{1}$$

Find at least one continuous spatial symmetry explicitly, i.e. construct the map $Q_n(s,t)$.

(ii) What is the content of Noether's theorem and how might it be useful?

Stage 2 *(Kepler problem)* Discuss:

(i) Review schematically with which steps one can reduce the Kepler problem (section 2.8) from 6 degrees of freedom for two objects moving in 3D to ultimately an equation for just a single dynamical variable (2.71).

(ii) In the lecture we had drawn the closed Kepler orbits quantitatively (i.e. with telling you how far away from the sun the planet is in a few selected places), but we had only drawn schematic sketches for the unbound orbits. Connect the unbound orbits with the Kepler Eq. (2.72), quantitatively (i.e. add axis labels to the drawings on page 44).

Stage 3 *(Celestial mechanics)* Look at this online simulator of the solar system.

(i) Understand what you see in terms of the results of the lecture material of week 5. Assuming circular orbits, find the orbital velocities of all the planets and compare with the animation.

(ii) Why can we understand the solar system based on week 5 at all, why don't we have to solve a 9 body problem?

(iii) Why do you think most of the comet orbits shown have much more elliptic orbits than planets and are tilted with respect to the ecliptic (solar system plane)? Or in other words, why do most planets have a fairly circular orbit?

Stage 4 PTO

(Symmetries again, advanced) DNA has a helical structure. In a massive over-simplification, we can hence take the interaction potential of another particle with DNA to have the form $V(r, a\phi + z)$ using cylindrical coordinates r, ϕ, z as shown in figure 1. The precise form of V does not matter in the following. The complete Lagrangian in cyclindrical co-ordinates is

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2) - V(r, a\phi + z). \tag{2}$$

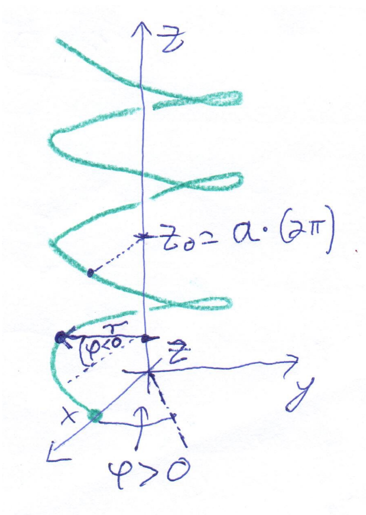

Figure 1: One of the strands of DNA forms a helix. In cylindrical coordinates you can describe the location of that helix by the conditions $r = r_0$, $a\varphi + z = 0$. Then after a length $z_0 = a(2\pi)$, the helix will have completed one cycle as shown. This should motivate that any interaction potential V of a particle with the helix can also be written as $V(r, a\phi + z)$.

(i) Identify a continuous symmetry $Q_n(s, t)$ based on the information given.

(ii) Use that to derive a conserved quantity using Noether's theorem.