PHY 305, I-Semester 2020/21, Tutorial 2

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss via online (video or audio) conference on a subchannel for your group.

Stage 1 *(math review)* Review from old course notes, books or the internet your knowledge about (ordinary, linear) differential equations. Make sure are comfortable with the answer to all these questions:

(i) What does an equation like $\dot{f}(t) = \frac{d}{dt}f(t) = g(t)$ tell me? What mathematical object is the solution of such an equation? What mathematical operation is in general required to solve such an equation and why?

(ii) For a differential equation $\frac{d^n}{dt^n}f(t) = g(t)$, how many unspecified constants does the solution contain and why? Which additional information can help to constrain those constants?

Stage 2 *(physics review)* Review from old course notes, books or the internet your knowledge about fundamental laws of physics. Make sure are comfortable with the answer to all these questions:

(i) A cycle is moving on a street with constant velocity such that its position in a street fixed reference frame is $x(t) = x_0 + v_0 t$. You are observing it from a train going parallel with velocity $v_1 \gg v_0$. What is the position in that cycle in your reference frame, which we assume is fixed on your seat in the train?

(ii) Why can the zero of energy be chosen arbitrarily?

(iii) Convert the following 2D cartesian coordinates into polar ones: $(x, y) = (0, 2)$, $(2, 0)$, $(1, 1)$, $(-3, -3)$, $(-2, 0)$, $(0, 0)$.

(iv) Convert the following 2D polar coordinates into cartesian ones: $(r, \varphi) = (4, \pi/4)$, $(2, \pi)$, $(3, 0)$, $(3, 8\pi)$, $(2, 3\pi/2)$, $(0, \pi/8)$.

Stage 3 (i) Think like a computer: How would you go about solving the differential equation $\dot{f}(t) = \frac{d}{dt}f(t) = g(t)$ by dividing time into small chunks of size Δt, thus allowing only discrete times t_0, $t_1 = t_0 + \Delta t$, $t_2 = t_0 + 2\Delta t$ etc...? Make a sketch of this computational solution versus the real solution.

(ii) You are throwing a stone very far, subject to gravity and friction (but not wind). How many dimensions does the relevant phase space have? Make a sketch of the stone's trajectory in the (x, z), (x, p_x) and (z, p_z) phase space planes (gravity along z).

(iii) Show that for a conservative potential, such that $\mathbf{F} = -\boldsymbol{\nabla} V(\mathbf{r})$ we have $W_{12} = V(\mathbf{r}_1) - V(\mathbf{r}_2)$.