PHY 305, I-Semester 2020/21, Assignment 5

Instructor: Sebastian Wüster
Due-date: 31.10.2020

(1) Weighted dice: [**10pts**] Consider a game dice made with the intention of cheating, such that it is heavier on the bottom. We place the origin $O = (0, 0, 0)$ on one corner of the cube, such that the cube extends from $0 < x < a$, $0 < y < a$ and $0 < z < a$. The density shall be $\rho(\mathbf{r}) = \rho_0 \left(1 - \frac{z}{2a}\right)$, reflecting that the dice is not regular.

(1a) Find all components of the inertia tensor with respect to above coordinates and origin.

(1b) Diagonalize the inertia tensor and find the moments of inertia about the principal axes and the principal axes *(You may use e.g. Mathematica)*

(1c) Explicitly (numerically) evaluate the principal axes as unit vectors and compare them and the moments of inertia with those of the the regular cube given in the lecture. Discuss (guess) similarities and differences of the rotation behavior of the regular and cheating dice.

(2) Stability of rotation: [**5pts**] Assume you have a rigid body with $\lambda_3 > \lambda_2 > \lambda_1$. Show that rotation around the body axes $\mathbf{e}_1$ and $\mathbf{e}_3$ will be stable, but around $\mathbf{e}_2$ not. *Hint: Stability implies that a small perturbation away from that axis will remain small, instability that it will grow (typically exponentially).*

(3) Sceptre on the ground: [**15pts**] Consider a monarch's golden sceptre as shown in figure 1, which has fallen on the ground and is now circling around the contact point of the handle and the ground with angular velocity Ω as shown.

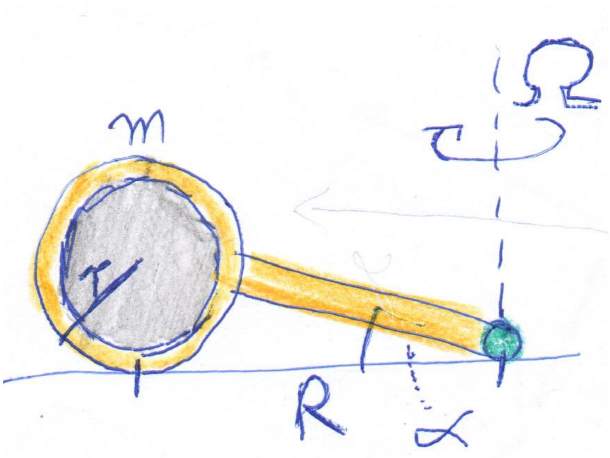

Figure 1: We model the golden sceptra as a <u>hollow</u> sphere of mass m and radius r. The length of the handle (ignore its width) is such that the contact point of the sphere with the ground is a distance R from the contact point of the handle with the ground, as shown.

(3a) If the sceptre circles without slipping, find the angular velocity vector that describes its rigid body motion.

(3b) Find the normal force between the ground and the sceptre.

(4) Inertia tensor: [**10pts**] You can see from many examples in books that finding moments of inertia analytically for complicated objects can be challenging. One way that always works, is numerical integration. A very simple method for numerical integration, is simply approximating the integral by a bunch of rectangles and writing $\int_a^b f(x)dx \approx \sum_k f(x_k)\Delta x$, where x_k are a set of equally spaced points between a and b with spacing Δx.

The template `Assignment5_program_draft_v1.m` allows you to generate quite complicated rigid bodies assembled out of spheres, cuboids or cylinders, but starts with a predefined "snow-figure". It should then numerically find the moment of inertia tensor (matrix) and diagonalize that matrix (again numerically) to show you the principal axes. The script is long due to lots of visualisations. You may ignore most of it unless interested.

(4a) The first element of the moment of inertia tensor I_{xx} is already implemented. Implement the other ones and run the script (you can search for `POSITION 3` in the script to find the place).

(4b) If you change the number of elements in the rigid body from 5 to 3 (`POSITION 1`), it will skip arms and nose. How does this affect the principal axes? Why?

(4c) Now play a bit with the rigid body/figure, by roughly understanding how it is defined (`POSITION 2`) and moving objects a bit by changing their centre (keep all coordinates within $-1\cdots 1$). If you reduce the symmetry of the figure, what happens to the principal axes? How does the magnitude of the moments of inertia reflect the shape of the object?