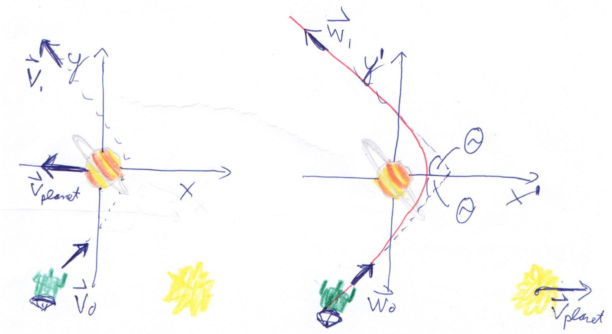
PHY 305, I-Semester 2020/21, Assignment 4

Instructor: Sebastian Wüster
Due-date: 03.10.2020

(1) Gravity assist: [10pts]
When sending space-probes out of the solar system (Pioneer, Voyager), humanity has used a trick called "gravity-assist" to help the probes reach solar escape velocity. As sketched in the figure below, for this the space-probe does a close fly-by near a heavy outer planet such as Saturn, changing its travelling direction while doing so.

Let the velocities in a sun fixed reference frame be $\mathbf{v}_0$ (probe in) $\mathbf{v}_1$ probe out and $\mathbf{V}_{\text{planet}}$ for the planet. Consider the situation where in a frame fixed on the planet the probe comes in on a hyperbolic Kepler orbit with eccentricity ϵ, such that the asymptotic trajectory far from the planet makes an angle θ with the x-axis, as shown. We denote the velocities in this frame by $\mathbf{w}_{0,1}$.



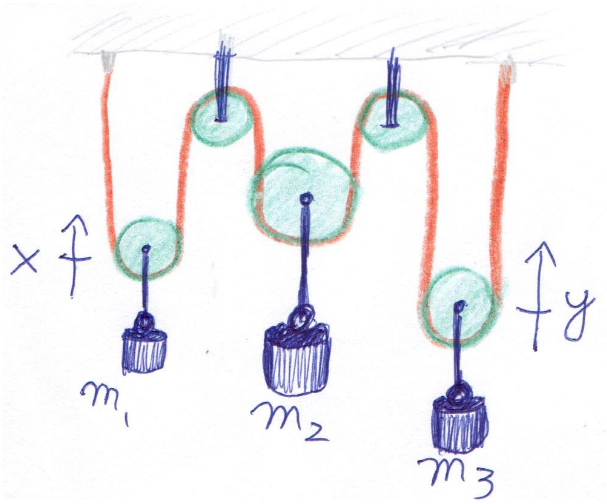
left: (left) Space probe fly-by past Saturn in a solar system fixed frame. (right) The same in a planet fixed frame. The sun is drawn for orientation in the bottom right corner, but not important in this question.

(1a) Show that $\theta = \pi - \arccos(-1/\epsilon)$.

(1b) Using this, show that in the solar system frame $v_1^2 - v_0^2 = 4w_0 V/\epsilon > 0$, where e.g. $v_0 = |\mathbf{v}_0|$ etc. This means the space probe is accelerated by its fly-by past the planet. Where does the required energy come from?

(1c) What happens when the direction of planet motion is opposite to that shown in the figure (with all other data identical)?

(2) Ropes and Pulleys: [10pts] Consider the arrangement of a single rope, threaded through five pulleys shown below. It is rigidly attached to the ceiling but each pulley can freely roll within the rope. Three weights are hung on the lower pulleys as indicated, with $m_1 = 5m_0$, $m_2 = 3m_0$, $m_3 = 7m_0$, where m_0 is some reference mass.



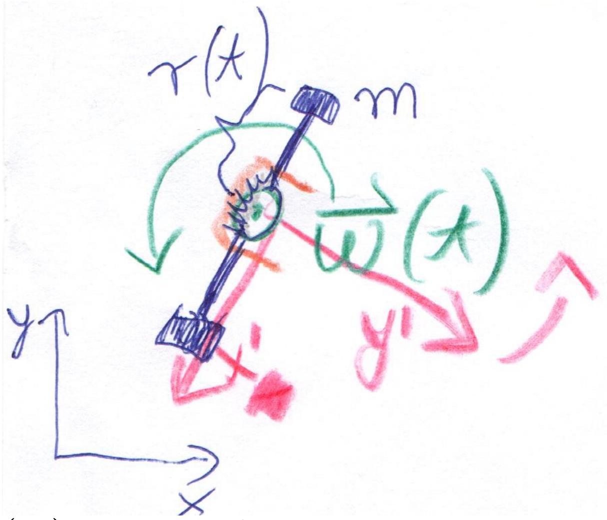
left: Sketch of rope, weight and pulleys as described in the text

(2a) Setup the Lagrangian in the generalised coordinates given by x and y, which are the upwards motion of weights m_1 and m_3 relative to their initial position.

(2b) Find at least one continuous symmetry of that Lagrangian and then use Noether's theorem to identify the associated conserved quantity.

(2c) Derive Lagrange equations and from those verify explicitly that the quantity is indeed conserved.

(3) Swivel-chair: [**10pts**] Watch the following videos related to angular momentum conservation (video1 , video2) and refer to the diagram below.



left: Sketch of professor (blue) on swivel chair (brown), viewed from the top. We also show the lab-frame (blue) and co-rotating frame (red). We assume the professor to be massless but carrying heavy weights of mass m in each hand. The distance from hands to rotation axis is $r(t)$.

(3a) Using angular momentum conservation (why can you use it?), find the rotation velocity $\Omega(t)$ (green) as a function of the initial rotational velocity $\Omega_0 = \Omega(t=0) > 0$ and the initial distance of the weights from the body $r_0 = r(t=0) > 0$ and the unspecified function $r(t)$.

(3b) In our derivation of the fictitious forces in a rotating coordinate system in section 2.9.2 (Eq. 2.87), we had assumed that the rotation axis does not depend on time.

Generalize the derivation for the case where $d\mathbf{\Omega}/dt \neq 0$. Give an equation for the additional fictitious force that you find.

(3c) Now rederive the result of (3a), using fictitious forces in the frame fixed on the person on the chair (see figure).

(4) Solar system simulator: [**10pts**] Formulate Newton's equations for all the major bodies in the solar system. The template `Assignment4_program_draft_v1.m` solves these for the subset (sun, earth, mars) in two dimensions (x,y).

(4a) Find the initial conditions that place earth and mars on the correct orbit (see internet) and verify them, using `assignment4_solarsystem_slideshow_v1.m`, for example by checking for the correct orbital period.

(4b) With the script `assignment4_radialevolution_draft_v1.m`, you can check the time evolution of the radial distance of mars from the sun. Edit the script so that panel (b) plots the effective radial potential $V_{\text{eff}}(r)$ and the energy of mars and explain the evolution of the radial distance with it. Then make Mars's orbit more eccentric, by reducing its initial velocity with an arbitrary scale factor. What happens?

(4c) [Bonus] Change the simulator to a triple-star system by changing earth and mars masses to equal that of the sun. What happens? Can you engineer a "stable" triple-star system?