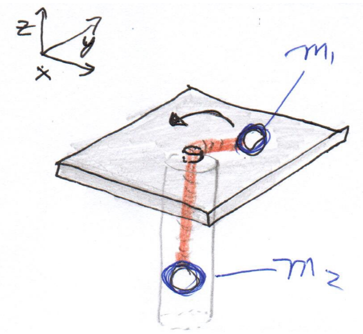
PHY 305, I-Semester 2020/21, Assignment 3

Instructor: Sebastian Wüster
Due-date: 21.09.2020

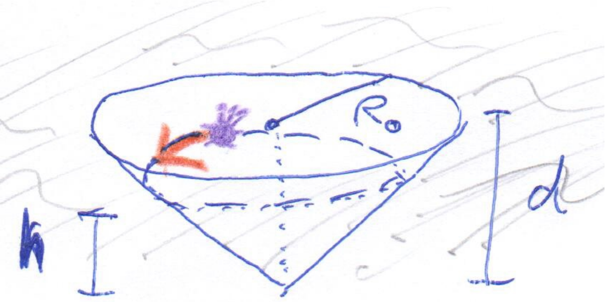
(1) Constrained system: [**10pts**]



left: A ball of mass m_1 can move without friction on a surface (grey) which forms the xy plane. It is connected through a rope which is threaded through a hole in the surface with a mass m_2 hanging below the surface. The latter is confined in a plexiglass tube, so that it can only move along the z direction. Consider only cases without slack in the rope and it being straight on the table, as in the figure.

(1a) Explicitly write the constraint equations for this problem, then discuss how many degrees of freedom the problem has and which generalised coordinates you would propose.
(1b) Write the Lagrangian for this problem.
(1c) Find the Lagrange equations, identify conditions where mass m_2 can be at rest, and find the frequency of small oscillations around these conditions.

(2) Alien fun: [**10pts**] Europa is a moon of Jupiter with a surface made of ice. An asteroid impact has made a perfectly shaped impact crater as shown in the figure below. An alien life form living on Europa wants to entertain itself by using the crater as a slide, frictionlessly sliding as shown in the figure such that it remains a constant height h over the bottom of the crater, despite Europa's gravity with surface acceleration g_E acting on the alien.



left: Sketch of impact crater (blue). It has a radius R_0 at the top and a total depth of d, in between the radius shrinks linearly. The alien is violet, sliding in a direction ini

(2a) Setup the Lagrangian in useful generalised coordinates, and find the Lagrange equations.
(2b) What is the frequency ω with which the Alien will complete its circles?
(2c) [Bonus question, i.e. no mark deduction of skipped] If it slightly misses the height h, what will be the frequency of oscillations around that height h?

(3) Double pendulum: [**10pts**] Find the Lagrangian for the double pendulum shown in the lecture notes, and from that the Lagrange equations.

(4) Bead on a spinning hoop: [10pts] Read again example 13 of the lecture notes.

(4a) Then implement the equation of motion using the same techniques as for assignment 1 Q4, into the same template code.

(4b) Plot the dynamics for some different initial angles $\theta(t = 0)$, for $\omega > \omega_c$, or $\omega < \omega_c$, include positions very close to the equilibrium angles θ_0 in your initial choices. Discuss your results.

(4c) A realistic bead would experience sliding friction on the hoop. Assume this is manifest in the equation of motion by an additional damping term $\ddot{\theta}(t) = -\gamma\dot{\theta}(t)$, and rerun simulations from some arbitrary initial conditions. What do you find?