PHY 305, I-Semester 2020/21, Assignment 1

Instructor: Sebastian Wüster
Due-date: 31st Aug 2020

(1) Block sliding on a wedge: [**10pts**] A pink cube of mass m slides on a wedge without friction as shown in Fig. 1. The wedge itself has mass M and angle α and is in turn sliding on a frictionless floor. All are at rest at $t = 0$. If the cube was initially at a height h, at what time will it reach the bottom? Solve the problem using Newtonian mechanics.

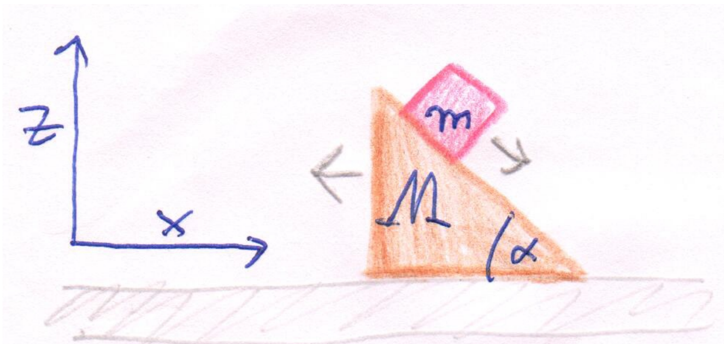

Figure 1: Cube, sliding on wedge, where the wedge is sliding on the floor.

(2) Two-dimensional harmonic oscillator: [**10pts**] A two-dimensional harmonic oscillator of mass m is one which has two-degrees of freedom x and y, and with a potential energy (see also Fig. 2)

$$V_{\text{pot}}(x, y) = \frac{1}{2} m \left(\omega_x^2 x^2 + \omega_y^2 y^2\right). \tag{1}$$

(2a) Using the definition of the gradient, write down Newton's equation for this oscillator explicitly in terms of x, y.
(2b) From your result in (a) and your knowledge of the 1D harmonic oscillator, find all conserved quantities, discuss how the dynamics will look like. What is the difference between this 2D harmonic oscillator and two separate 1D harmonic oscillators?
(2c) Find the time-averaged kinetic energy and potential energy separately for the motion related to x. How are they related? *Hint: The time-average of a periodic quantity* $E(t)$ *is* $\bar{E} = \int_0^T E(t)dt/T$, *where* T *is the period.*

(3) Line integrals: [**10pts**] The harmonic oscillator from the previous questions is being moved from position $\mathbf{r}_1 = (x_1, y_1) = (1, 0)$ to position $\mathbf{r}_2 = (x_2, y_2) = (0, 3)$ in a straight line in 2D, as shown in the figure below. Calculate the work exerted on the oscillator using the explicit line integral over the force in two dimensions. Which would be a much simpler way to arrive at the same answer?

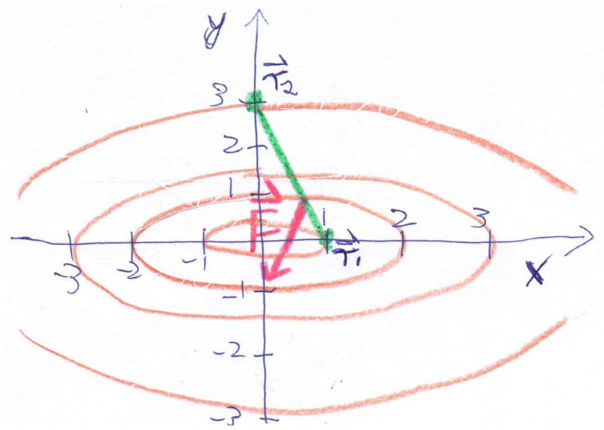

Figure 2: Sketch of 2D harmonic oscillator potential energy Eq. (1) (we show brown equal energy contours, for some exemplary values of ω_k), resultant force (pink arrow) and curve of motion for finding the work (green).

(4) Computational question: harmonic oscillator, damping driving and modulation: [10pts] Newton's equation for a point mass m in one-dimension subject to a force $F(t)$ is $m\ddot{r} = F(t)$.

(4a) Computational algorithms typically need us to convert second order differential equations in time into a system of first order differential equations in time. By using both, the position r and the velocity v as variables, do this conversion, i.e. find a coupled system of first order differential equations that is equivalent to Newton's equation.

(4b) Write these equations for the case of a 1D simple harmonic oscillator, where $F_k = -kx$ and implement them in the template file `Assignment1_program_draft_v1.m`. Solve the equations for a couple of different choices of parameters, and analyse your result using `Assignment1_plot_oscillator_v1.m` and `Assignment1_plot_phasespace_v1.m`, discuss.

(4c) Now we extend the calculations to add a friction force $F_f = -\gamma v$ and a driving force $F_d = F_0 \sin(\omega t)$. Again analyze your result and compare it with solutions for the damped, driven harmonic oscillator that you can find in textbooks or the internet. (Use some arbitrary dimensionless units throughout question 4, e.g. mainly pick parameters in the range $[0, 5]$).