Week ⓪

PHY 305 Classical Mechanics

Instructor: Sebastian Wüster, IISER Bhopal, 2020

These notes are provided for the students of the class above only. There is no guarantee for correctness, please contact me if you spot a mistake.

0 Administrative affairs

(i) Office: drop me an email to arrange a skype call
Office hours: Wednesdays 3 pm - 6 pm.
Phone: 1213
Email: sebastian@iiserb.ac.in
webpage: http://home.iiserb.ac.in/∼sebastian/teaching.html

(ii) Literature:

- J. R. Taylor, Classical Mechanics, *"Classical Mechanics"* [TT]
- H. Goldstein, C. Poole and J. Safko, *"Classical Mechanics"* [GPS]
- L. Landau and E. Lifshitz, *"Mechanics"* [LL]
- L. Hand and J. Finch, *"Analytical Mechanics"* [HF]

The course will mainly follow TT and GPS. Where stray topics are taken from elsewhere I will try to indicate this.

(iii) Lectures and tutorials:

- Lectures will be provided as online video, based on explanation of these notes, possibly with additional movies or presentation elements. Please download and view these early in the week, to be prepared to tackle assignments.
- I am arranging lecture notes into "week" segments, based on similar content. While most of those segments should indeed take a week for us to work through, this will not be true for all and they may take less or more often more time.
- There will be two weekly online sessions (i) One will be alternating between two different formats: (A) In one week we shall have a flipped classroom tutorial, conducted in the format of a parallel video chat session paired with examineer life-quiz and a assignment-class where we discuss solutions to the assignments. In these we will do little tasks to understand the material more deeply. You will have to have gone through the lecture notes *prior* to the flipped classroom session. (B) The following week we shall do a TA-class/assignment class in which we discuss the solutions of assignments together. (ii) The second weekly session will be a Q&A chat, where you can ask me or the TA questions on anything from the lecture, tutorials or assignments.

(iv) Assessment:

- **Four scheduled Quizzes with examineer: 10+10+20+10=50%** There will be four quizzes lasting 30min-1.5 hours, conducted using the examineer webpage . These are "open notes" quizzes, so make your notes available offline or on a second device beforehand. The third of these quizzes will weigh 20% and can be considered "mid-sem" quiz.
- **Assignments: 20%** There will be about seven assignments handed out with a two week deadline each. I expect you to form teams of 3-5 students and stick in these teams for the semester. Hand in only one solution per team. The TA is instructed to give full marks for *any serious attempt* at a given question of the assignment, even if the result is wrong. This is to discourage copying and encourage doing it yourself. Additionally however the TA is asked to deduct marks for messy presentation and blatant copying from anywhere. I expect you to meet up with your team-members via video-chat to discuss the assignments. Submit your final solution via email to the TA. This may be a good opportunity to learn LaTex and nicely typeset your solution, but handwritten and good-quality scanned/photographed is fine too. A
- **Numerics component of assignments:** Moderns science almost always necessitates the heavy use of computers. Most assignments will contain a numerics component, to be done using matlab. Please make the campus license version downloadable from the CC webpage available on at least one computer in your team. You shall also need VPN access to the campus network for the matlab license. For each assignment, I will provide a template code package that you have to only minorly edit. See notes on numerics assignments online. No prior experience of either programming or matlab should be required, but if you read some online notes regarding introduction to matlab in the first weeks, that might allow you to have more fun with this part of the assignments.
- **TA class and flipped classroom attendance: 10%** This will be compulsory, we record the sessions as thus see if you were present. When attendance is less than 60% the marks for this component are 0, and for more than 90% they are 100. In between they are as per fractional attendance.
- **Final exam online: 20%** The exams will try to test understanding of the essential *physics* concepts taught, not maths. For guidance regarding what are the most important concepts look at the quizzes and assignments. All exams will be designed to give a significant advantage to those students that solved all assignments by *themselves* within their team. Exams will likely be conducted online using examineer and shall make heavy use of examineer's randomisation features to render teamwork ineffective.

0.1 Course Outline

1) Motivation and Review of Newtonian Mechanics: ∼ 2 weeks
   • Why do we need more powerful method than Newton's equations? State of the art research in classical mechanics. Fundamental concepts of Mechanics.

2) Lagrangian Mechanics: ∼ 5 weeks
   • Variational Principle, Lagrange equations, Generalized coordinates, Constraints, Conservation laws and symmetries, Noether's theorem, two-body problem.

3) Motion of Many Bodies and Rigid Bodies: ∼ 4 weeks
   • Oscillations and normal modes, Inertia tensor, Euler angles, Top, precession.

4) Hamiltonian Mechanics: ~ 3 weeks
• Hamilton's equations and phase-space, Poisson Brackets, canonical transformations, Hamilton-Jacobi theory.

0.2 Math content

Moving from year 2 to year 3 at IISERB and into your physics major, you will experience a signifiant upwards jump of the level of mathematics in most courses, certainly in this one. I will expect you to be familiar with the following math topics:

- vectors, scalar-products, cross-products
- matrices, eigenvalues and eigenvectors of matrices
- Solutions of ordinary differential equations
- complex numbers
- basic differentiation and integration
- Dirac delta-function

Where you have doubts about being sufficiently familiar with any of those, please consult your math course notes, books or online resources. Focus on books/courses of the kind "mathematics for scientists and engineers" that tell you "how to actually get calculations done". An exemplary online resource is e.g. this link

For the following required math tools, I shall attempt to give a small self contained introduction:

- non-cartesian / curvilinear coordinate systems
- differentiation and integration in higher dimensions, line integrals
- calculus of variations

1 Motivation, Newtonian Mechanics

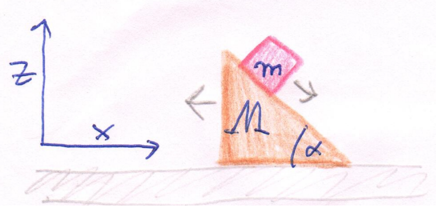
1.1 Motivation

I know Newton's equation. Why is that not enough?

- In principle we can solve all mechanics problems with Newton's equations. However these can quickly become cumbersome for certain problems. Examples of such problems are (i) those involve kinematic constraints (see section 2.4), (ii) those favouring non-cartesian coordinates such as motion on the surface of a sphere.

- In these cases you will see that problems can typically be much more efficiently tackled in two advanced formulations of mechanics that we shall learn in this course: Lagrangian mechanics (due to Lagrange 1788) and Hamiltonian mechanics (due to Hamilton 1835). These are fully equivalent to Newtonian mechanics (due to Newton 1685).

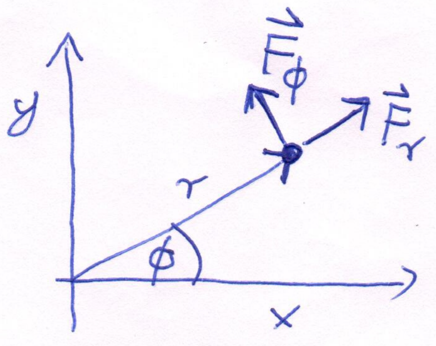
Example 1, Block sliding on a wedge:



left: Sketch of Block sliding on a mobile wedge without Friction.

Consider the inclined plane above. It is more fancy than the usual one, in that the wedge forming the inclined plane can itself move on a frictionless surface. How long will it take the block to reach from top to bottom of the wedge? You will treat this problem in assignment 1 to freshen up you Newtonian mechanics. We will then see in section 2.5 how it is much neater solved using Lagrangian mechanics.

Example 2, Newton's equations in polar co-ordinates:



left: Sketch of point mass described in polar coordinates

Consider an object moving in two-dimensions, which you describe in polar coordinates. In those coordinates, Newton's equations $\mathbf{F} = m\mathbf{a}$ take the form

$$F_r = m\left(\ddot{r} - r\dot{\phi}^2\right),$$

$$F_\phi = m\left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right). \tag{1.1}$$

We will see why and review polar coordinates in section 1.4.6, but you can already see that Eq. (1.31) are cumbersome due to the mixing of different derivatives of r and ϕ variables.

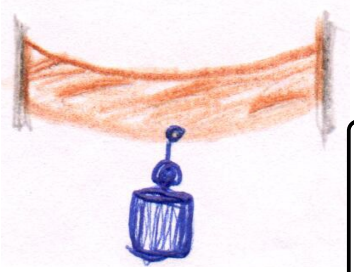
- The advanced formulations also introduce many useful conceptual tools that form the basis of quantum mechanics and quantum field theory, e.g. you will learn that "the Hamiltonian" was in fact a classical concept long before it became a quantum one, and that there is a classical analog to a commutator called a "Poisson bracket".
- Finally, there are also surprisingly many open research questions even in classical mechanics. One of the most prominent arena with these is non-linear dynamics and chaos.

Constraints

- Motion may be constrained e.g. to 1D

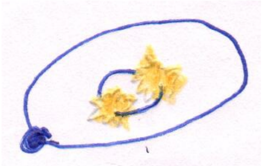
Continuum-mechanics

- Mechanics of extended deformable object



Orbital Mechanics

- Clean "point masses".
- Many-body problems, stability.



Legendre Transformation

- Mechanics $H = p\dot{q} - L$
- Thermodyn $dU = TdS - PdV \leftrightarrow dH = TdS - VdP$

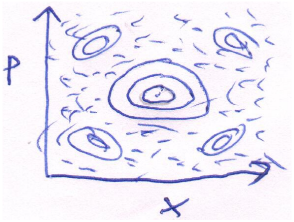
Classical Mechanics

- Three equivalent formulations
  1. Newtonian mechanics
  2. Lagrangian mechanics
  3. Hamiltonian mechanics
- Concepts: (generalized) Force, (canonical) momentum, angular momentum, energy, work, inertia (tensor), configuration-space, phase-space, equilibrium, stability, symmetries, conservation laws

The bubble diagram here gives an idea of the central role played by "classical mechanics" throughout all of modern physics.

Chaos theory

- Nonlinear systems can be chaotic.
- Motion becomes practically unpredictable

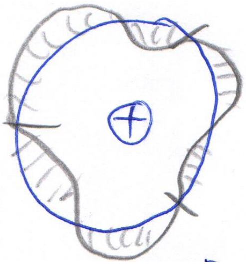


Quantum mechanics

- Formal structures based on Hamiltonian mechanics
- $H \leftrightarrow \hat{H}$
- $\sum_j \frac{du}{dq_j}\frac{dv}{dp_j} - \frac{du}{dp_j}\frac{dv}{dq_j}$ $\leftrightarrow [\hat{x},\hat{p}]$
- $m\frac{d}{dt}\langle\hat{x}\rangle = \langle\hat{p}\rangle$

Atomic physics

- Bohr orbit model $\leftrightarrow$
- Hamilton-Jacobi action-angle theory



Quantum Field theory

- Lagrangian $\leftrightarrow$ Lagrange density $L = \int d^3x \int dt \mathcal{L}(\mathbf{x}, t)$
- Noether theorem

1.2 Research frontier

- Given classical mechanics is ~ 400 years old, one might be tempted to assume all is known about it. This is far from true, particularly in the two boxes "continuum mechanics" and "chaos theory" on the previous page.
- Some interesting examples are shown in the movie recording of these notes, thanks to Dr. Deepak Kumar who gave this course last year. The following are links to the movies contained on those slides:
  Euler's disk
  Shear in granular material
  Soft Robot

1.3 Limitations of classical mechanics

While we have seen that classical mechanics successfully describes a bewildering array of physical phenomena, you also know already that it miserably fails in two important regimes:

- $v \sim c$: When the speed of objects approaches the speed of light, we have to use special relativity. That dramatically changes mechanics, but still relies on many of the same fundamental principles as classical mechanics, such as momentum and energy conservation.
- $E \sim \hbar\omega$, or $d \sim \lambda_{\mathrm{dB}}$: When approaching atomic distance and energy scales (and in other cases), the quantisation of physical observables becomes important and we have to use quantum mechanics. Again, a lot of quantum mechanics is just classical mechanics in disguise.

Technically "relativistic quantum mechanics" is always right, so we could just attempt to understand all phenomena using that. Practically however, for many-many important research topics results would show no noticeable difference to the classical ones and would be 1000% times harder to obtain (in most cases impossible to obtain).