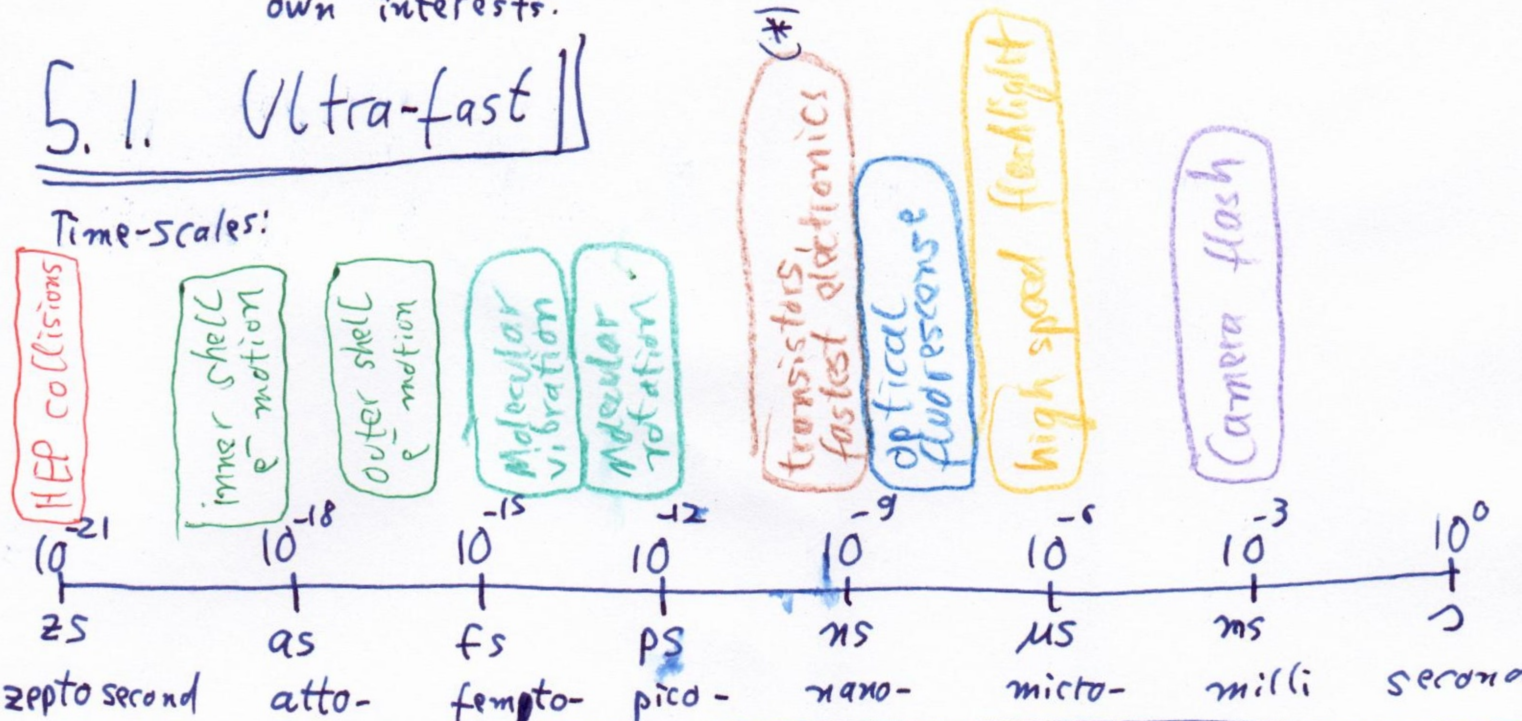
(13)

5. Frontiers of modern AMO physics

We can only cover a small selection, biased by my own interests.

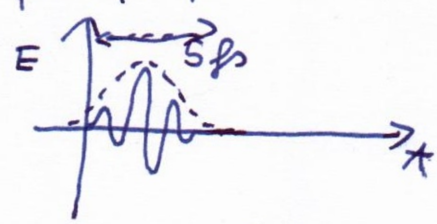
5.1. Ultra-fast

Time-scales:



- To resolve e.g. vibrational molecular motion, need ~fs light pulses.
- Consider visible, e.g. 500nm $\Rightarrow \nu = 6\cdot 10^{14}$ Hz $\quad \Delta T = 1$ fs

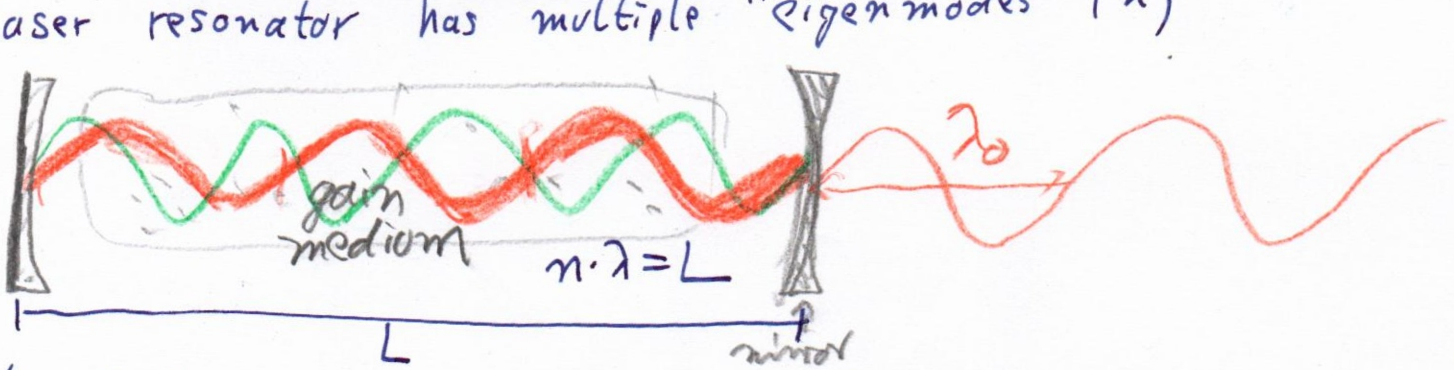
  $\Delta T \cdot \nu \sim 0.6$ a few fs pulse, has only few optical cycles:



- Hard to generate, can't switch on-off laser (*)

5.1.1. Frequency Combs / Femtosecond lasers

Laser resonator has multiple "eigenmodes" (n)

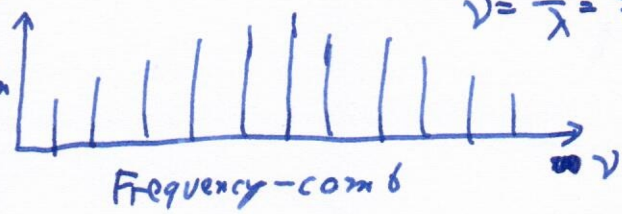


- Normal laser, want "single-mode" operation (one λ)
- If we use multi-mode (mode-locked) Laser:

Frequency spectrum of mode locked laser:

$$\tilde{E}(\omega) = \underbrace{\sum_{n}}_{N\text{-modes}} c_n \, \delta(\omega - \omega_n)$$

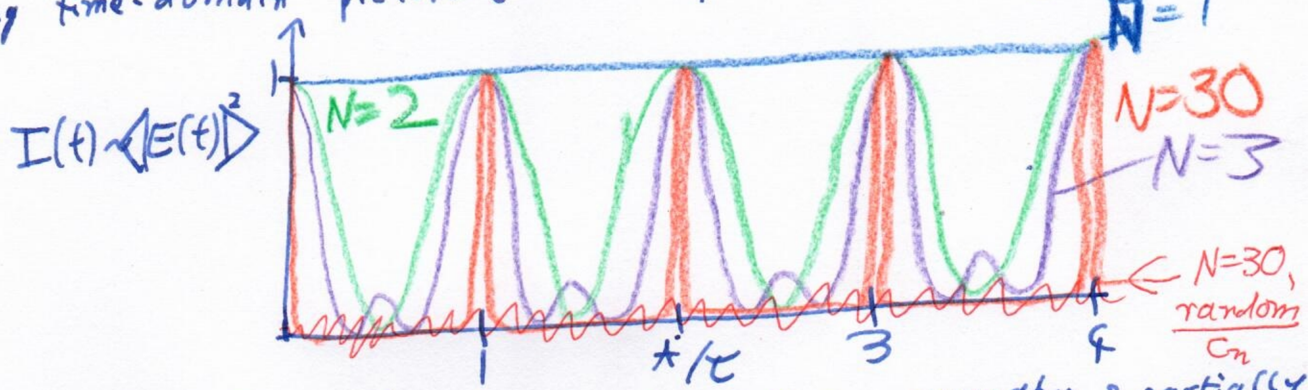
$\nu = \frac{c}{\lambda} = \frac{n}{\tau}$



(4.32)

τ = round-trip time $[2L/c]$

Resulting time-domain picture: (c_n = const, $N = 1, 2, 3, 30 \ldots$)

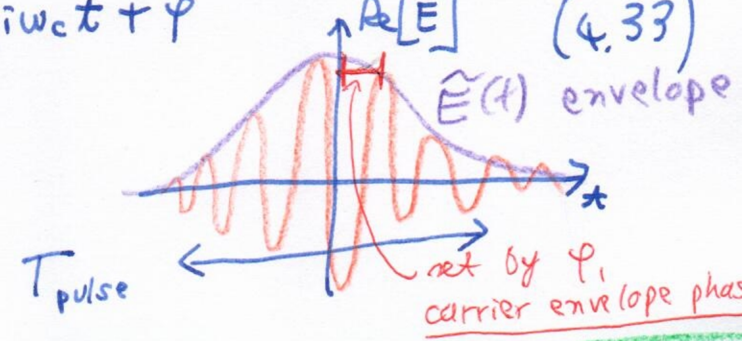


Interpretation: Short pulse is bouncing back & forth in resonator & partially coupled out each time. ⇒ fs pulse <u>train</u>

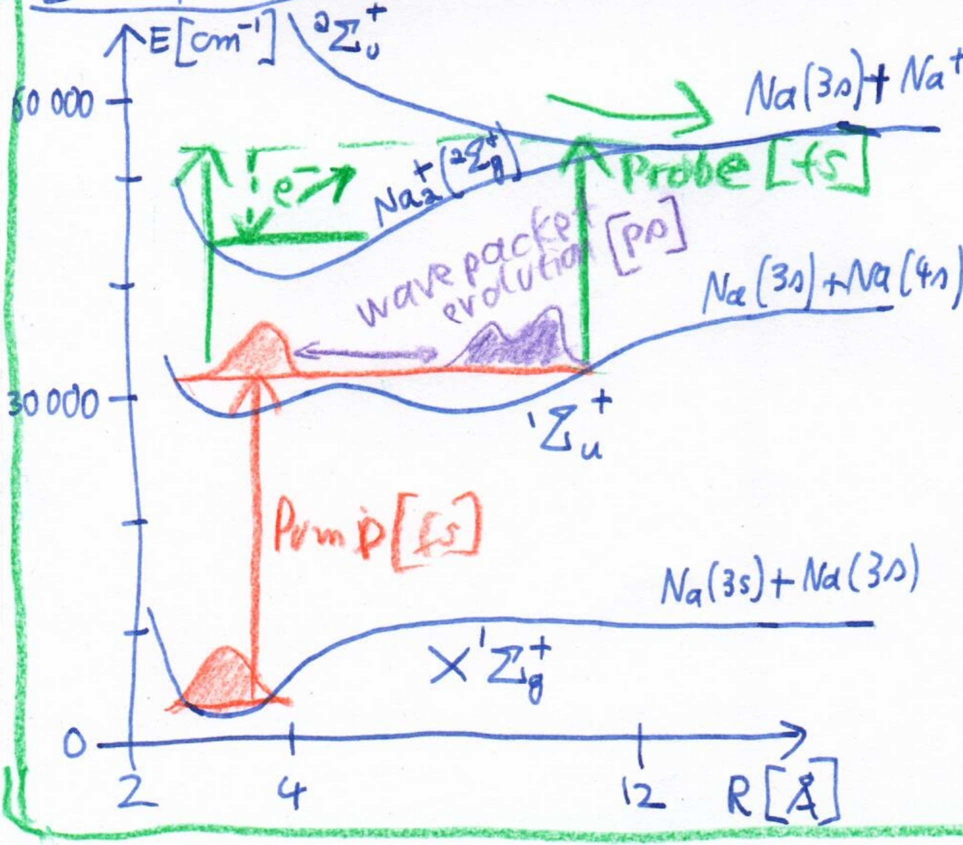
<u>single Pulse-waveform:</u>

$$E(t) = \tilde{E}(t)\, e^{i\omega_c t + \varphi}$$

(4.33)

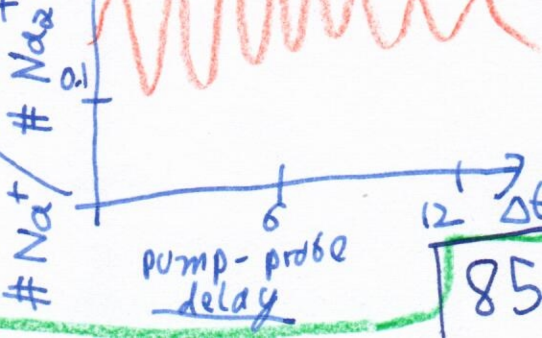


Example 5.1.1) Femtosecond pump-probe spectroscopy (of Na_2)



- After pump, vibrational wavepackets evolves in excited elec state
- Depending on nuclear separation $R(t)$, probe puls can dissociate Na_2, or not:

<u>Signal:</u>



5.1.2. Strong fields, tunnel ionization

- Another consequence of the laser focussing all its energy into very short pulse: Very high intensities.
- In terms of (3.7), $\hat{H}'(t)$ may no longer be perturbative, $E(t)$ may become comparable to Coulomb-field of nucleus!

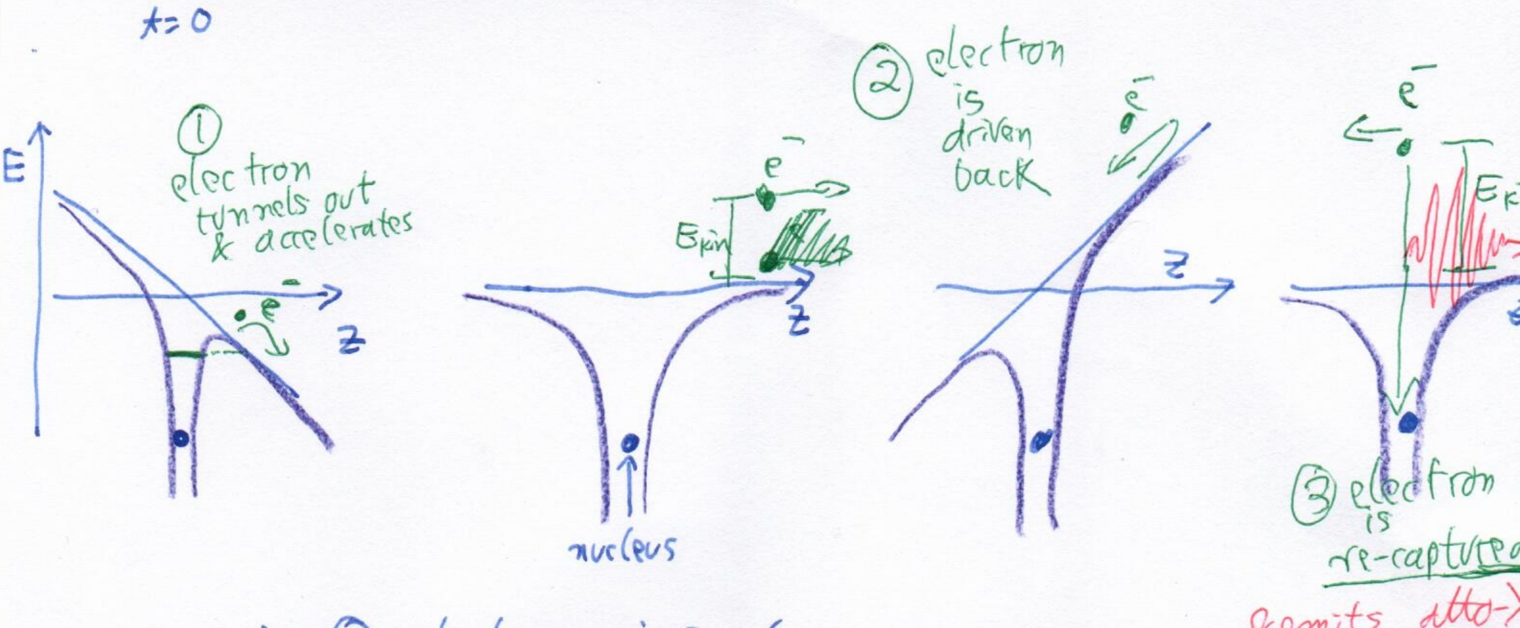
Example: Electric field in hydrogen at $r = a_0$ is $E(a_0) = \frac{Z e^2 m_e}{(4\pi\varepsilon_0)^2 \hbar^2}$

Continuos laser intensity $I = \varepsilon_0 |E(a_0)|^2 c$

Assume tight $\underbrace{\approx 1\mu m}_{r_{foc}}$ focus: power $P = I \cdot (\pi r_{foc}^2) \sim 2$ GigaWatt

Pulsed operation $P' = P \cdot \frac{T_{pulse}}{\tau} \sim$ few 100 W

Three-step model of HHG (high-harmonic generation)



Through ①⇔②, electron gains ~

$$\boxed{\text{ponderomotive energy} \quad U_p = \frac{e^2 E_a^2}{4 m \omega_0^2} \quad \text{(cycle averaged kin energy in field} \sim E_a \cos(\omega_0 t))} \qquad (4.3$$

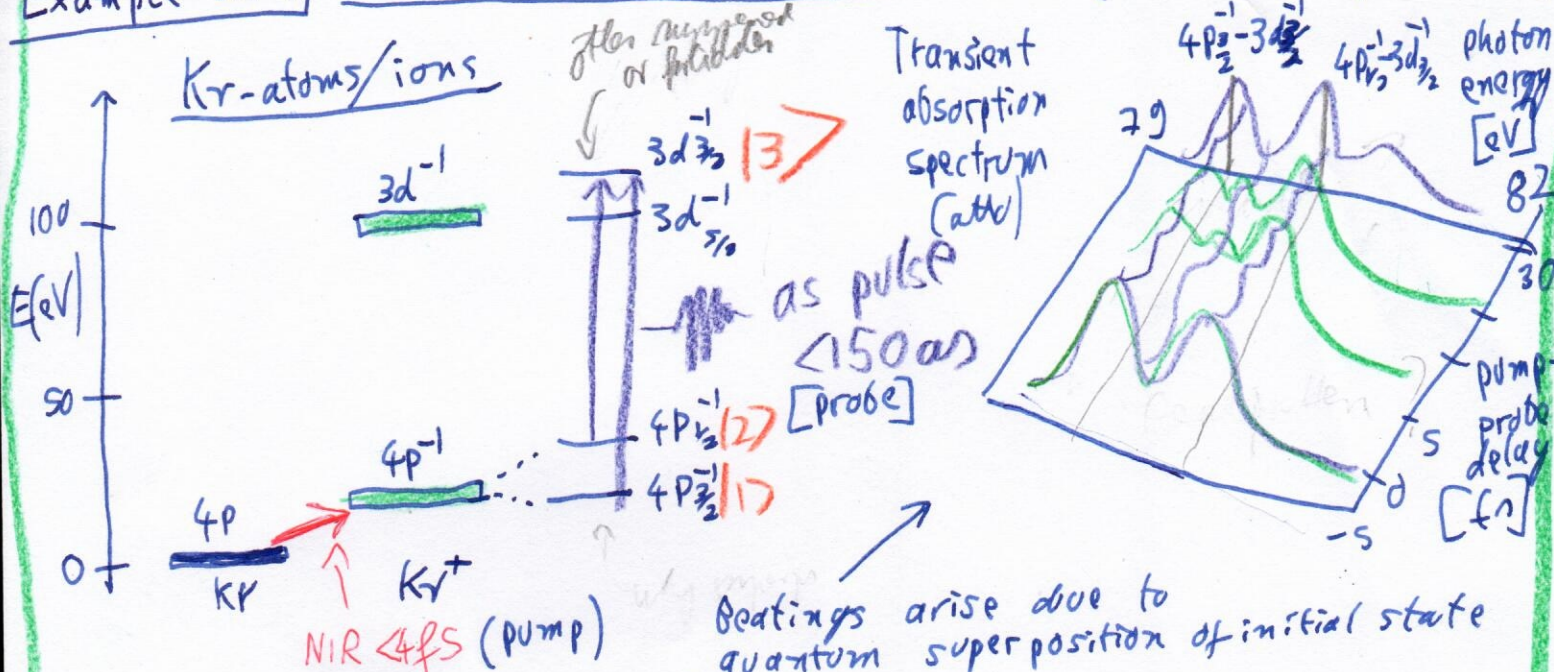
Duration of emitted pulse ~ time-scale of electron motion.

- Classical electron orbit $Z=1$ (outer shell) $t_{orb} \approx 150$ as

  $Z = 20$ (inner shell) $t_{orb} = 7$ as

↳ Can in this way (HHG) generate <u>attosecond laser pulses</u>.

Example 5.1.2. Real-time valence electron motion (Nature 466 (2010) 739)



Beatings arise due to quantum superposition of initial state

(Neglecting all details of atto absorption)

$$|\psi(0)\rangle \sim |1\rangle + |2\rangle$$

$$|\psi(t)\rangle \sim e^{-iE_1 t/\hbar}|1\rangle + e^{-iE_2 t/\hbar}|2\rangle$$

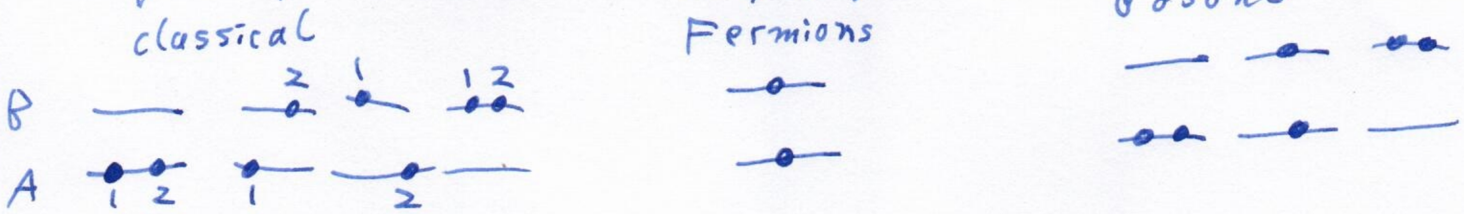
$$|\langle 3|\hat{d}|\psi(t)\rangle|^2 = |\langle 3|\hat{d}|1\rangle e^{-iE_1 t/\hbar} + \langle 3|\hat{d}|2\rangle e^{-iE_2 t/\hbar}|^2 = |\langle 3|\hat{d}|1\rangle|^2 + |\langle 3|\hat{d}|2\rangle|^2 + \langle 3|\hat{d}|1\rangle^* e^{i(E_1-E_2)t/\hbar} + c.c.$$

5.2. Ultra-cold

We have discussed in section 3.2.3 how thermal motion of atoms, or pressure broadens spectral lines.
Since manipulations (excitation, trapping) are more controlled for narrow lines, it is appealing to cool atomic gases as much as possible.

5.2.1. Bose-Einstein Condensation

For indistinguishable quantum particles (1.2.6), need to change way we count [# possible many-body states]



This gives rise to quantum-statistics: Bose-Einstein ⊖ / Fermi-Dirac ⊕ distribution function

$$n_i = \frac{1}{e^{(E_i-\mu)/k_B T} \pm 1}$$

for mean number of particles in a state with energy E_i (4.35)

- μ is the chemical potential (fixes $N = \sum_i n_i$)
- classical (Boltzmann) distribution, no ± 1
- BE-distrubution diverges for groundstate $(E_i - \mu) \to 0$ ⇒ treat separately
- Find below a certain temperate, need macroscopic occupation of groundstate:

BEC transition temperature

$$k_B T_C \approx (\hbar\omega) N^{\frac{1}{3}}$$

- N particles
- 3D harmonic trap $V(\vec{x}) = \frac{1}{2} m\omega^2 |\vec{x}|^2$ (4.36)

e.g. $N = 10\,000$, $\omega = (2\pi)\, 100\,\mathrm{Hz}$ $\quad T_c = 97\,\mathrm{nK}$

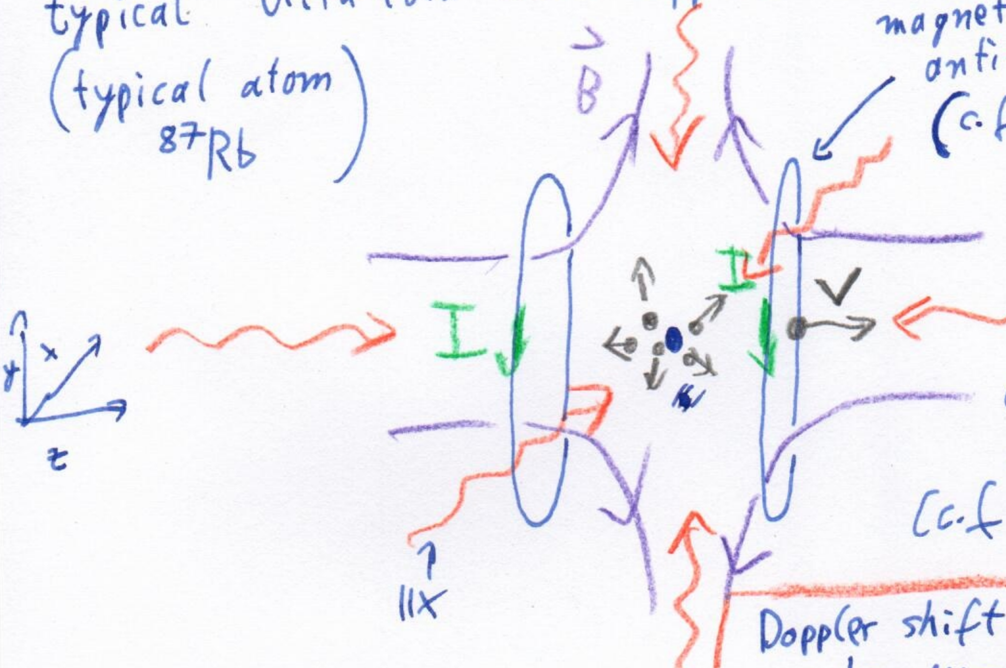
Groundstate / Condensate fraction (4.37)

$$\frac{N_0(T)}{N} = \left[1 - \left(\frac{T}{T_c}\right)^3\right]$$

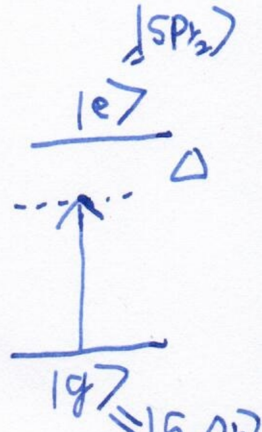
5.2.2. Laser cooling and trapping

typical ultra-cold atom apparatus
(typical atom ^{87}Rb)

magnetic trap, anti-Helmholtz coils (c.f example in section 2.2)



cooling laser detuned [c.f section 3.14]



(c.f example 2.1)

Doppler shift
$\omega' = \omega - \boxed{v} k$
shifts only <u>fastest</u> atoms into resonance!
(cooling through recoil)

- Can combine magnetic field shifts on $|g\rangle, |e\rangle$ to get spatially dependent <u>optical force</u> $\Rightarrow$ MOT (<u>m</u>agneto-<u>o</u>ptical-<u>t</u>rap)
- Laser-cooling can reach $\simeq 100\,\mu K$ (Doppler limit)
- Improved laser cooling (Sisyphus cooling)

<u>Recoil Limit:</u>

$$k_B T = \frac{(\hbar k_{las})^2}{2M} \approx 0.1 - 1\,\mu K$$
(4.38)

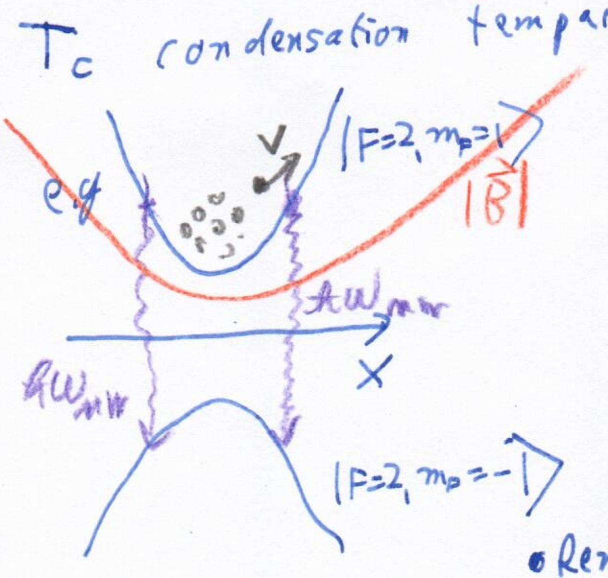
- To finally reach T_c condensation temparature:

<u>Evaporative cooling</u>

Magnetic trap:

$E = \pm g_L \mu_B m_F |\vec{B}|$

(Eq (2.23))



- Drive microwave transition ω_{mw} to anti-trapped $m_F=-1$ state at large $|\vec{x}|$
- Loose only <u>most energetic</u> atoms
- Looses most atoms
- Remainder $T \simeq nK$

5.2.3. Condensate mean-field

- Prior to condensation, Many-body wavefunction $\psi(\vec{R}_1, \dots, \vec{R}_N)$ for N atoms. Intractable for N say $N \simeq 10\,000$
- After condensation, can describe using condensate wave-function / order-parameter / mean-field $\phi(\vec{R})$ which obeys

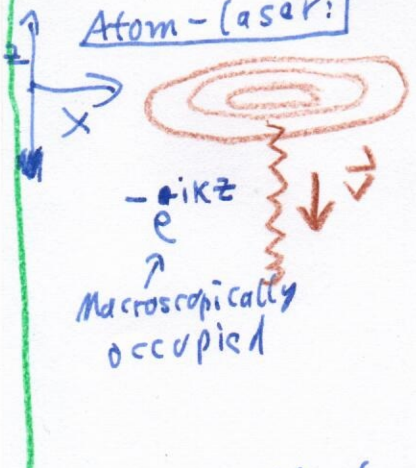
Gross-Pitaevskii-equation:

$$i\hbar \dot{\phi}(\vec{R}) = \left[-\frac{\hbar^2}{2m} \vec{\nabla}^2 + V(\vec{x}) + U_0 |\phi(\vec{R})|^2 \right] \phi(\vec{R}) \qquad (4.39)$$

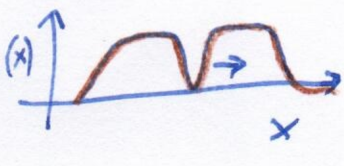
- where $V(\vec{x})$ is the trapping potential.
- $U_0 = \frac{4\pi \hbar^2 a_s}{m}$ describes atomic collisions
- a_s = s-wave scattering length (see QM, partial-wave treatment of scattering)
- $n(\vec{R}) = |\phi(\vec{R})|^2$ atom-density, $N = \int d^3\vec{R}\, |\phi(\vec{R})|^2$

Example: BEC phenomena:

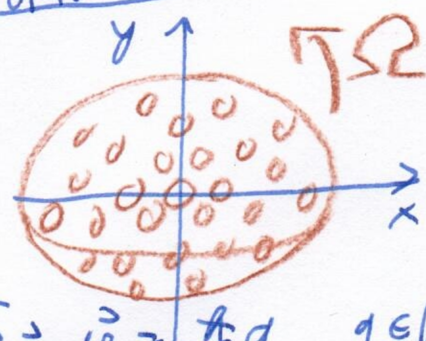
Atom-laser:



Solitons

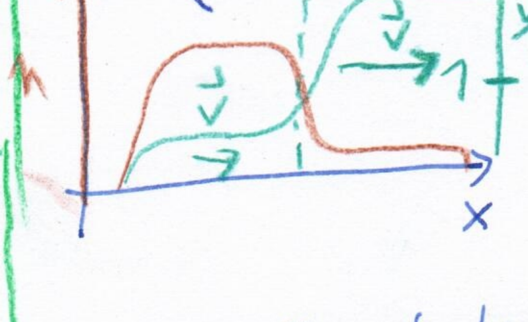


Vortex-lattice



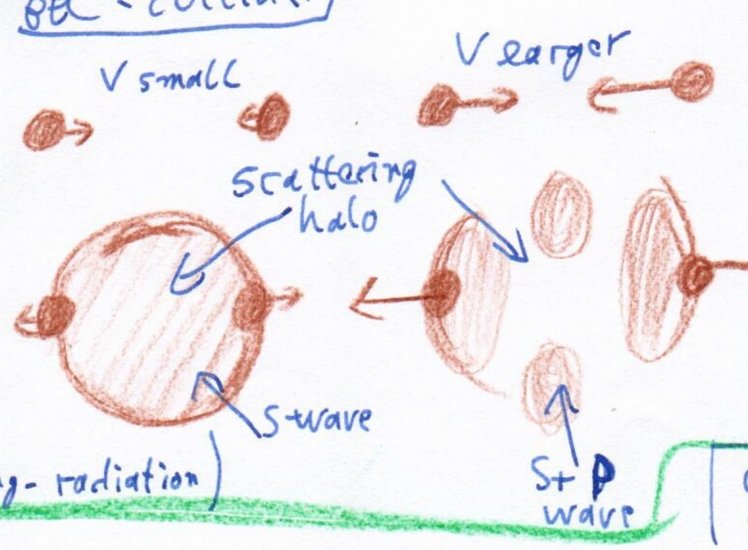
$\oint_C \vec{v} \cdot d\vec{l} = \frac{\hbar}{m} q \qquad q \in \mathbb{N}$

sonic black-holes



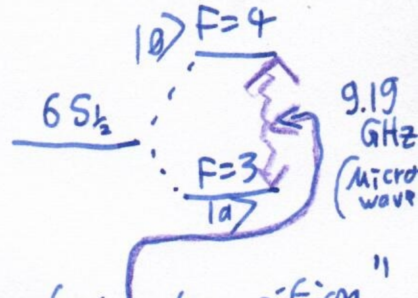
thermal phonon emission (Hawking-radiation)

BEC-collider



5.3. Atomic clocks

^{133}Cs atom, nuclear spin $I = \frac{7}{2}$, hyperfine splitting of $6S_{1/2}$

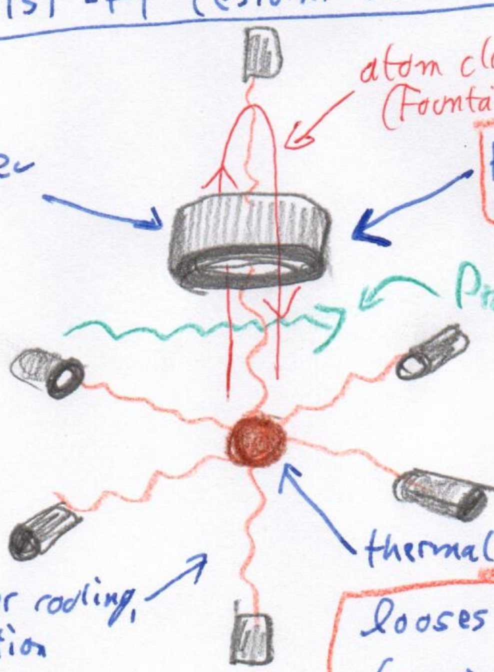


Definition of SI-unit "second"

„Duration of 9 192 631 770 periods of radiation for transition."

Example: NIST-F1 cesium Fountain clock:

www.nist.gov



Fine-tune on F=3 → F=4 transition by maximizing final N_{atoms} in $|b\rangle$

↳ ω_{mw} ⇝ time

looses <1s in 100 million years (precision $3\cdot 10^{-16}$)

5.4. Quantum-simulation

Seen for many e^--atoms/molecules: Many-body quantum mechanics extremely challenging.

For N particles that can be in $\boxed{M}$ states each, dimension of Hilbertspace

$$d \sim M^N$$

- quickly too large to solve on classical computers

Idea by R. Feynman

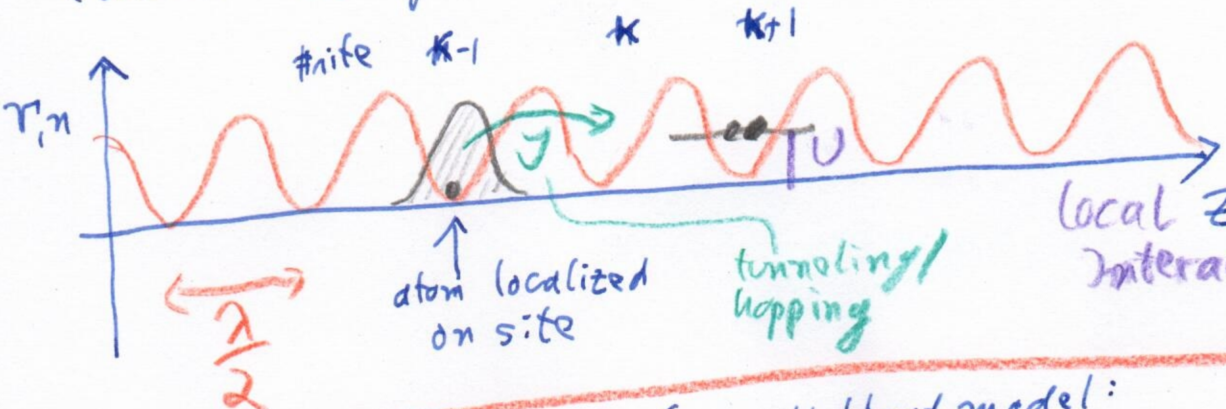
„Find another quantum-system with mathematically equivalent Hamiltonian, where all parameters are under experimental control"

↳ (experimental) analogue quantum simulator

5.4.1. Bose-Hubbard model

Consider Bose-gas in optical lattice (= standing light wave)

Nature 415 (2002) 39

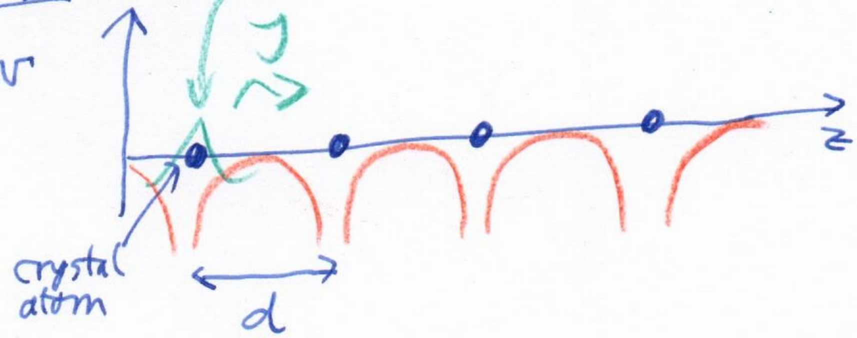


Many-body Hamiltonian / Bose-Hubbard model:

$$\boxed{\hat{H} = -J\sum_{k}\left(\hat{a}^{+}_{k+1}\hat{a}_{k} + \hat{a}^{+}_{k-1}\hat{a}_{k}\right) + \frac{U}{2}\sum_{k}\hat{n}_{k}(\hat{n}_{k}-1)} \quad (4.40)$$

- $\hat{a}_{k}$ $(\hat{a}^{+}_{k})$ destroys (creates) a Boson at site n
- $\hat{n}_{k} = \hat{a}^{+}_{k}\hat{a}_{k}$ is the number operator.

Quantum simulator for Hubbard model in condensed matter physics:



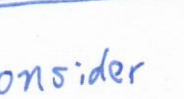
- electrons in a metal crystal (then fermionic)
- here d, J, U cannot be changed (for a given material)

- In cold atoms, J, λ, U can all be tuned.

5.4.2. Strongly interacting Rydberg systems

Consider Rydberg atoms $|nlm\rangle$ $n \gg 20$ (see section 2.1.4)

Two Rydberg atoms (~ Rydberg molecule but not bound), interact via Van-der-Waals interactions (see sec. 4.5).

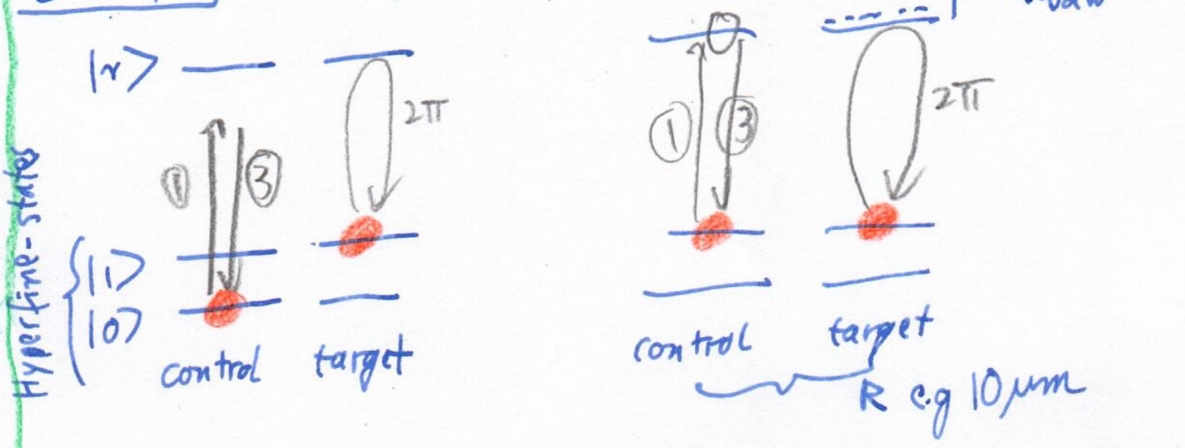


$V_{VdW}(R) \sim \frac{C_6}{R^6}$ $\quad C_6 \sim n^{11}$

- ground-state range $\sim 20\, a_0 \sim 0.01\,\mu m$
- Rydberg range $\sim 10\,\mu m$

Example: Rydberg CNOT Gate

$V_{vdW}(R)$



Nature Phys. 5 (2009) 110 & 115

requires optically resolved addressing
$\therefore R \gtrsim 2\mu m$

Sequence:

(i) Rabi-π pulse on $|1\rangle \leftrightarrow |r\rangle$ transition for <u>control only</u>

Case: (control ↓ ↓ target)

$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
↓			
$\|00\rangle$	$\|01\rangle$	$\|r0\rangle$	$\|r1\rangle$

(ii) Rabi 2π pulse on $|1\rangle \leftrightarrow |r\rangle$ transition for target only

↓	↓	↓	↓
$\|00\rangle$	$-\|01\rangle$ (see Eq. 3.24)	$\|r0\rangle$	$\|r1\rangle$

(iii) Rabi π-pulse on $|1\rangle \leftrightarrow |r\rangle$ <u>control only</u>

↓	↓	↓	↓
$\|00\rangle$	$-\|01\rangle$	$\|10\rangle$	$\|11\rangle$

Truth table:

$|00\rangle \to |00\rangle$
$|01\rangle \to -|01\rangle$
$|10\rangle \to |10\rangle$
$|11\rangle \to |11\rangle$

Controlled Z-gate
↳ can get C-Not from here

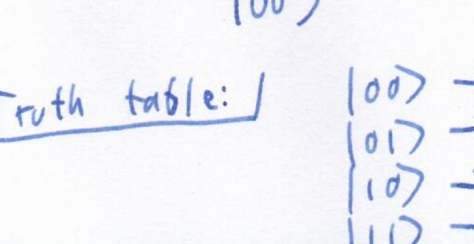
Other examples:

long-range spin chains

Nature Phys. 12 (2016) 1095

energy transport

Phys Rev Lett 114 (2015) 113002



Huge-range surfaces

molecular

Phys. Rev. Lett 106 (2011) 153002

The End