Topology II (MTH 516/616)

Mid-Semester Examination (22/02/2019)

Time: 10 AM - 12 PM Total Marks: 30

Problem A. Attempt **three** questions. Each question carries 4 marks.

(1) Let X be a path connected space and $f : X \to X$ be a map. Show that $H_0(f) : H_0(X) \to H_0(X)$ is the identity map.
(2) Let $m < n$ be positive integers. Show that every continuous map $f : S^m \to S^n$ is null-homotopic.
(3) Let $f : S^n \to S^n$ be an antipodal map. Show that $deg(f) = (-1)^{n+1}$
(4) Let $f : |K_1| \to |K_2|$ is a map and $\phi : K_1 \to K_2$ be a simplicial map. Let ϕ is a simplicial approximation to f. Show that if $f(\alpha) \in \sigma$ then $|\phi|(\alpha) \in \sigma$, where σ is a simplex in K.

Problem B. Attempt **two** questions. Each question carries 6 marks.

(1) Let K be the Simplicial complex in $\mathbb{R}$ with vertices $0, 1/2$ and 1 and edges $[0, 1/2]$ and $[1/2, 1]$, so that the underlying space $|K|$ is the closed unit interval $[0, 1]$. Let $f : [0, 1] \to [0, 1]$ be the map $f(x) = x^2$. Prove that $f : |K| \to |K|$ does not have a simplicial approximation $\phi : K \to K$. Using Simplicial approximation theorem we know that there exist a $\phi : sd^n(K) \to K$ which is simplicial approximation to f. For above f find out ϕ and n which gives Simplicial approximation to f.
(2) Let K be a simplicial complex. Suppose that there exists a vertex w of K with the property that, If vertices $v_0, v_1, \cdots, v_q$ span a simplex of K then so do $w, v_0, v_1, \cdots v_q$. Show that $H_0(K) = Z$, and $H_q(K)$ is the zero group for all $q > 0$. Hint: Define map $C_q(K) \to C_{q+1}(K), (v_0, v_1, \cdots, v_q) \mapsto (w, v_0, v_1, \cdots v_q)$.
(3) Let $m, n \geq 0$ be non-negative integers. Choose base points $x_0 \in S^m$ and $y_0 \in S^n$ and let $S^m \wedge S^n$ be the one-point union formed with respect to these base points. Compute $H_i(S^m \wedge S^n; S^n)$ for all $i \geq 0$.

Problem C. Attempt **one** question. Each question carries 6 marks.

(1) Show that the Euler characteristics of a compact, path-connected, triangulable topological group is zero.
(2) Let U and V are two subsets of S^n and $f : U \to V$ is a homeomorphism. If U is open then show that V is open.

Note: S^n denote n-sphere. You can use any homology theory definition as per your need.