TOPOLOGY II (MTH 516/616)

MID-SEMESTER EXAMINATION (29/04/2019)

Time: 9.30 AM - 12.30 PM Total Marks: 40

Problem A. Attempt **five** questions. Each question carries 3 marks.

(1) Give a counterexample to show that $H_n(X, A)$ is not necessarily isomorphic to $H_n(X/A)$.
(2) Show that Projective space $\mathbb{R}P^n$ is CW complex.
(3) Show that $\pi_i(X \times Y) \cong \pi_i(X) \times \pi_i(Y)$ for all $i \geq 1$.
(4) Let $f : X \to Y$ be a n-sheeted covering space. Show that $\chi(X) = n\chi(Y)$.
(5) Suppose that (X, x_0) is a space with a universal cover (Y, y_0), with covering map $p : Y \to X$ such that $p(y_0) = x_0$. Then there is a canonical isomorphism between $\pi_2(X, x_0)$ and $H_2(Y)$.
(6) Let X is a CW complex with one 0-cell and all other cells in dimensions $> n$. Show that $\pi_i(X, *) = 0$ for $i < n$.
(7) Let X be an topological space. Show that,

$$\chi(X) = \sum_i (-1)^i dim(H^i(X, \mathbb{Q}).$$

Problem B. Attempt **five** question. Each question carries 5 marks.

(1) Show that the Euler characteristics of a compact, path-connected, triangulable topological group is zero.
(2) Let $f : S^n \to S^n$ be a map. If f is not homotopy equivalence, Show that f has a fixed point. For every integer m, find f such that $deg(f) = m$.
(3) Let X be a closed orientable homology n-manifold. Show that $H_{n-1}(M)$ is a free abelian group.
(4) Compute higher homotopy group $\pi_i(S^1)$ for all i. Also show that $\pi_i(S^n) = 0$ for all $0 < i < n$.
(5) Let M be a manifold and x be a point in M. Show that for all i,

$$H_i(M, M - \{x\}) = \begin{cases} 0 & \text{if } i \neq 0 \\ \mathbb{Z} & \text{if } i = n \end{cases}$$

(6) Compute the Cohomology ring of the projective n-plane $\mathbb{R}P^n$.