Topology II (MTH 516/616), Assignment-2

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Problem 1. Find example a compact connected subset of $\mathbb{R}^2$ that is not a polynedron.

Problem 2. For each vertex $v \in Vert(K)$, prove that $st(v)$ is an open subset of $|K|$ and that the family of all such stars is an open cover of $|K|$.

Problem 3. Let $v_0, v_1 \cdots v_q \in Vert(K)$. Prove that $\{v_0, v_1 \cdots v_q\}$ spans a simplex of K if and only if $\cap_{i=0}^{q} st(v_i) \neq \emptyset$.

Problem 4. If $\phi : K \to L$ is a simplicial approximation to $f : |K| \to |L|$, then $|\phi|$ is homotopic to f.

Problem 5. Let $K^{(q)}$ is the set of all simplexes $s \in K$ with $dim(s) \leq q$, show that $K^{(q)}$ is subcomplex of K. If $\phi : K \to L$ is a simplicial map. then $\phi(K^{(q)}) \subset L^{(q)}$ for every q. Conclude that $Image(\phi) \subset L^{(dim(K))}$

Problem 6. Compute the oriented simplicial homology of S^n, T^2, P^2, K^2.

Problem 7. Show that for simplicial complex K and L. Set of homotopically equivalent map from $|K| \to |L|$ is countable.

Problem 8. If nonempty open sets $U \subset \mathbb{R}^m$ and $V \subset \mathbb{R}^n$ are homeomorphic, then show that $m = n$.

Problem 9. Let K be a simplicial complex satisfying the following conditions: (i) K has no simplexes of dimension greater than n; (ii) Every $(n-1)$-simplex of K is a face of exactly two n-simplexes; (iii) For any two n-simplexes σ and τ of K, there exists a finite sequence of n-simplexes, beginning with σ and ending with τ, in which each adjacent pair of simplexes have a common $(n-1)$-dimensional face. Show that $H_n(K)$ is either $\mathbb{Z}$ or the trivial group, and that in the former case it is generated by a cycle which is the sum of all the n-simplexes of K, with suitable orientations.

Problem 10. Let X be a polyhedron. Show that $X \times S^n$ is also polyhedron. What about product of two polyhedron?

Problem 11. Let K and L be simplicial complexes in $\mathbb{R}^n$. Show that $K \cap L$ is a sub complex of K and L, but it is not true in general that $K \cup L$ is a simplicial complex.

If $|K \cap L| = |K| \cap |L|$ then show that $K \cup L$ is also simplicial complex.

Problem 12. If σ is a simplex and L is a line containing an interior point of σ , then $\sigma \cap L$ is a closed interval and $L \cap bd(\sigma)$ consists of the two end points.