# Commutative Algebra (MTH-518/618)

Quiz (28/08/2017)

Time: 60 minutes Maximum Marks: 10

**Attempt all questions. Use separate page for each answer.**

**Problem 1.**

(1) Let $A$ be a ring, $S$ a multiplicative subset, and $M$ a $A$-module. Show that $M = S^{-1}M$ if and only if $M$ is an $S^{-1}A$-module. (2)

(2) Suppose $I$ is an ideal of $A$ and $p$ is a finitely generated prime ideal. Show that $\sqrt{I} = p$ if and only if there exists an $n > 0$ such that $p^n \subseteq I \subseteq p$. (2)
Where $\sqrt{I}$ denote radical of $I$.

**Problem 2.** Show that $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n \cong \mathbb{Z}_{gcd(m,n)}$. (3)

**Problem 3.** Let $A$ be a ring, $m \subset \mathcal{J}(A)$ an ideal, where $\mathcal{J}(A)$ is the Jacobson radical of $A$. Let $\alpha, \beta : M \to M$ be two maps of finitely generated modules. Assume $\alpha$ is surjective and $\beta(M) \subseteq mM$. Define $\sigma := \alpha + \beta$. Show that $\sigma$ is an isomorphism.

*OR*

Let $A$ be a local ring, $M$ and $N$ finitely generated $A$-modules. Prove that if $M \otimes N = 0$ then $M = 0$ or $N = 0$. (3)