Commutative Algebra (MTH-518/618)

Mid Semester Examination (16/09/2017)

Time: 120 minutes Maximum Marks: 30

Attempt all questions. Use separate page for each answer.

Problem 1: Solve any three. Each question carries 3 marks.

(1) Let A be an ring and I be an ideal of A. Show that if I is free as an A-module then I is principal.

(2) Let A be a Noetherian ring and $I \subset A$ a radical ideal. Then, there exist prime ideals $p_1, \cdots, p_m$ with

$$I = \cap_i \, p_i.$$

(3) If $A[X]$ is Noetherian, is A necessarily Noetherian ?

(4) Let A be an ring and I be an ideal. Define $Ass(I)$ and show that in a Noetherian ring $Ass(I)$ is a finite set. (9)

Problem 2: Solve any three. Each question carries 5 marks.

(1) Calculate all prime and maximal ideals in $(\mathbb{R}[X])$.

(2) Show that in an Artinian ring, every prime ideal is maximal.

(3) Calculate all radical ideals in $\mathbb{Z}$. Also compute $Ass(I)$, where I is an ideal in $\mathbb{Z}$.

(4) Let $f : A \to B$ be a homomorphism of rings and let S be a multiplicatively closed subset of A. Let $T = f(S)$. Show that $S^{-1}B$ and $T^{-1}B$ are isomorphic as $S^{-1}A$-modules. (15)

Problem 3: Solve any two. Each question carries 6 marks.

(1) Define a ring $A := \mathcal{C}^0([0,1])$ set of continuous functions on the interval $[0,1] \subset \mathbb{R}$. Show that zero ideal is not primary ideal. Compute $Ass(< 0 >)$.

(2) Let M be a finitely generated A-module and $f : M \to A^n$ a surjective homomorphism where n is positive integer. Show that $ker(f)$ is finitely generated. (6)