MTH 518/618

COMMUTATIVE ALGEBRA

Assignment-4

Submission Date: 13/11/2017

Problem 1. Let A be a domian. Then A is a Dedekind domain if and only if every non-zero fractinary ideals of A is invertible.

Problem 2. Show that the rings of integers in an algebraic number field over $\mathbb{Q}$ is a Dedekind domain.

Problem 3. Let A be an Artinian ring whose nilredical is zero. Show that A has only finitely many ideals.

Problem 4. Let A, B be rings and $\phi : A \to B$ an integral ring extension. Then prove that,

$$dim(A) = dim(B).$$

Problem 5. Let A be a Dedekind domain with only finitely many prime ideals. Then A is a principal ideal domain.

Problem 6. Problem 3,4,5,7 from Chapter 9.

Problem 7. Note that class number of domain is cardinality of ideal class group. Compute the class number of following Integral domian.

(1) $\mathbb{Z}[i]$.
(2) $\mathbb{Z}[\omega]$, where ω is a cube root of unity.
(3) $\mathbb{Z}[-\sqrt{2}]$.

Text Book: Introduction to Commutative Algebra, Atiyah, Macdonald.