MTH 513/613: INTRODUCTION TO RIEMANNIAN GEOMETRY

COURSE INFORMATION

- **Instructor:** Dr. Sanjay Kumar Singh <sanjayks@iiserb.ac.in>
- **Office:** 210, Academic Building 1.
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- **Webpage:** http://home.iiserb.ac.in/∼sanjayks.

- **Class Time:** Mon Tues, Wed 2-3 PM .
- **Venue:** 108 AB-1.

Learning Objectives. A Riemannian manifold is a smooth manifold equipped with additional geometric structure called a Riemannian metric, and this structure provides a framework to measure geometric quantities such as length and angles on the manifold. Associated with a Riemannian metric are the fundamental concepts of a Riemannian connection, geodesics and curvature. First, the basic properties and results associated with these are studied. The course then explores the relationship between geodesics and curvature. After studying such questions (which are local in nature), the focus turns to global questions, and the course culminates in a study of certain important results concerning how curvature affects the topology of the manifold.

Syllabus: The official Course Syllabus is as given in the Course Contents booklet.

http://acad.iiserb.ac.in/pdf/mth.pdf

Course Contents:

- Review of differentiable manifolds: vector bundles, tensors, vector fields, differential forms, Lie groups
- Riemannian metrics: Definition, examples, existence theorem; model spaces of Riemannian geometry
- Connections: connections on a vector bundle, linear connections, covariant derivative, parallel transport, geodesics
- Riemannian connections and geodesics: torsion tensor, Fundamental Theorem of Riemannian Geometry, geodesics of the model spaces, exponential map, convex neighborhoods, Riemannian distance function, first variation formula, Gauss' lemma, geodesics as locally minimizing curves; completeness, statement of Hopf-Rinow Theorem

- Curvature: Riemann Curvature Tensor, Bianchi identity, scalar, sectional and Ricci curvatures
- Jacobi Fields: Jacobi equation, conjugate points, second variation formula, spaces of constant curvature (if time permits)
- Curvature and topology: Gauss-Bonnet Theorem, Bonnet-Myers Theorem, Cartan-Hadamard Theorem

Text Book:

- Manfredo P. do Carmo, Francis Flaherty, Riemannian Geometry: Theory & Applications (Mathematics: Theory & Applications), Birkhäuser 1992.

Reference Books:

- J. M. Lee. Riemannian Manifolds, An introduction to Curvature. Graduate Texts in Mathematics. Springer (1997).
- Gallot, D. Hulin, J. Lafontaine. Riemannian Geometry. Springer (2004).
- Chavel. Riemannian geometry, a modern introduction. Cambridge University Press (2006)
- S. Kobayashi, K. Nomizu. Foundations of differential geometry, vol. -I, Wiley Interscience Publication (1996).

Assignment. There will be 8 assignments in this course. The assignments will be posted on the Google Class course webpage.

Home work and class exercise. Every class you will get some homework you don't need to submit. You can discuss it with me.

Grading Policy: The grading policy for the 2023-24 1st Semester is divided into the following components

- Seminar/ Presentation/Assignments (20%)
- Quiz (10%)
- Mid Semester Examination (30%)
- Final Examination (40%)
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Quiz: There will be two quizzes in the semester. The average of both quizzes will be added.

- Quiz Date and Time: –/07/2023.
- Quiz Date and Time: –/09/2023

Office Hours: By appt.

Important Note: : Exam problems will be based on assignments, homework, and class exercises.

***.** In case of any further questions regarding the course, please email me.