

Assignment-3

(1) Let $\gamma: I \rightarrow \mathbb{R}^n$ be any curve. Show that a vector field V along γ is parallel with respect to the Euclidean connection if and only if its components are constants.

(2) Let ∇ be a linear connection (Affine) connection on M . Show that covariant differentiation along a curve γ can be recovered from parallel translation, by the formula following formula:

$$D_t V(t_0) = \lim_{t \rightarrow t_0} \frac{P_{t,t_0}^{-1} V(t) - V(t_0)}{t - t_0}$$

(3) A Riemannian manifold is said to be flat if it is locally isometric to Euclidean space.

Show that A Riemannian manifold is flat if and only if its curvature tensor vanishes identically.

(4) Compute the exp map for $M = \mathbb{R}^n, S^n, S^1$