AN INTRODUCTION TO RIEMANNIAN GEOMETRY (MTH 613)

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ASSIGNMENT-2

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Problem 1. . Let X, Y be two vector fields on $\mathbb{R}^3$ defined by

$$X(x_1, x_2, x_3) = (2x_3 - x_2)\frac{\partial}{\partial x_1} + x_1\frac{\partial}{\partial x_2} - 2x_1\frac{\partial}{\partial x_3}$$

$$Y(x_1, x_2, x_3) = x_3\frac{\partial}{\partial x_2} - x_2\frac{\partial}{\partial x_3}$$

- Calculate the Lie bracket $[X, Y]$.
- Let $S^2 = \{x \in R : ||x|| = 1\}$ be the standard unit sphere. Show that the restrictions of the vector fields X, Y to S^2 are vector fields on S^2.
- Check that the restriction of the Lie bracket $[X, Y]$ to S^2 is also a vector field on S^2.

Problem 2. Solve problem 1,2,8, 9, 10 from Page-77-85, Chapter 3..

Problem 3. Solve problem 4,5,6, 7 from Page-103-106, Chapter 4.

Text Book: Manfredo P. do Carmo, Francis Flaherty, Riemannian Geometry: Theory & Applications (Mathematics: Theory & Applications), Birkhäuser 1992.