MTH 508/608: INTRODUCTION TO DIFFERENTIABLE MANIFOLDS AND LIE GROUPS

COURSE INFORMATION

- **Instructor:** Dr. Sanjay Kumar Singh <sanjayks@iiserb.ac.in>
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- **Webpage:** http://home.iiserb.ac.in/∼sanjayks.
- **Class Time & Venue:** Tuesday and Thursday Wed 11-11.55 AM **108** AB-1.

  and Wednesday 12 to 12.55 PM **316**, AB-1.

Course Goal: The main goals of this course are:

- Understand manifolds and maps between them and learn how to construct them;
- Understand the symmetry groups of manifolds (Lie groups) and their infinitesimal versions (Lie algebras) and how to work with them;
- Study differential and integral calculus on manifolds using objects called differential forms

Syllabus: The official Course Syllabus is as given in the Course Contents booklet.

$$http://acad.iiserb.ac.in/pdf/mth.pdf$$

Course Contents:

- **Differentiable manifolds:** definition and examples, differentiable functions, the existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, Whitney embedding theorem
- **Vector fields:** vector fields, statement of the existence theorem for ordinary differential equations, one parameter and local one-parameter groups acting on a manifold, the Lie derivative and the Lie algebra of vector fields, distributions and the Frobenius theorem
- **Lie groups:** definition and examples, the action of a Lie group on a manifold, the definition of Lie algebra, the exponential map, Lie subgroups and closed subgroups, homogeneous manifolds: definition and examples
- **Tensor fields and differential forms:** cotangent vectors and the cotangent space at a point, cotangent bundle, covector fields or 1-forms on a manifold, tensors

on a vector space, tensor product, symmetric and alternating tensors, the exterior algebra, tensor fields and differential forms on a manifold, the exterior algebra on a manifold

- **Integration:** orientation of a manifold, a quick review of Riemann integration in Euclidean spaces, differentiable simplex in a manifold, singular chains, integration of forms over singular chains in a manifold, manifolds with boundary, integration of n-forms over regular domains in an oriented manifold of dimension n, Stokes theorem, definition of de Rham cohomology of a manifold, statement of de Rham theorem, Poincare lemma

Text Book:

- J. Lee, Introduction to smooth manifolds, Springer, 2nd Edition.

Reference Books:

- W. Boothby, An Introduction to differentiable manifolds and Riemannian geometry, Academic Press, 2002
- F. Warner, Foundations of differentiable manifolds and Lie groups, Springer, GTM 94, 1983
- M. Spivak, A comprehensive introduction to differential geometry, Vol. 1, Publish or Perish, 1999
- G. de Rham, Differentiable manifolds: forms, currents and harmonic forms, Springer, 1984
- V. Guillemin and A. Pollack., Differential topology, AMS Chelsea, 2010
- J. Milnor, Topology from the differentiable viewpoint, Princeton University Press, 1997
- J. Munkres, Analysis on manifolds, Westview Press, 1997
- C. Chevalley, Theory of Lie groups, Princeton University Press, 1999
- R. Abraham, J. Marsden, T. Ratiu, Manifolds, tensor analysis, and applications, Springer, 1988
- S. Kobayashi, K. Nomizu. Foundations of differential geometry, vol. -I, Wiley Interscience Publication (1996).
- Nigel Hitchin, 'Differentiable manifolds', Oxford lecture notes, 2014.
- Rob van der Vorst, 'Introduction to differentiable manifolds. free online lecture notes available at https://www.few.vu.nl/ vdvorst/notes-2012.pdf.

Assignment. There will be assignments in this course. The assignments will be posted on the Google Class course webpage with a submission deadline.

Home work and class exercise. You will get homework you don't need to submit in every class. You can discuss it with anyone or google it.

Grading Policy: The grading policy for this course is divided into the following components

- Seminar/ Presentation/Assignments (20%)
- Quiz (10%)
- Mid Semester Examination (30%)
- Final Examination (40%)

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Quiz: There will be two quizzes in the semester. The average of both examinations will be added.

- Quiz Date and Time: –/01/2024.
- Quiz Date and Time: –/03/2024

Office Hours: Feel free to meet and discuss with me after emailing first.

Important Note: : Exam problems will be based on assignments, homework, and class exercises.

***.** Schedule conflicts with other courses do NOT constitute valid excuses for exam and lecture time. You are supposed to resolve them before registering for the classes. If you have any further questions regarding the course, please email me.