Introduction to Algebraic Topology (MTH-507)

Mid Semester Examination (17/02/2018)

Time: 120 minutes Maximum Marks: 30

Attempt all questions. Use separate page for each answer.

Problem 1. Solve any five questions. (3+3+3+3+3)

(1) Find a continuous map $f : T \to S^1$ which is not homotopic to constant, where T is a torus.
(2) Find a continuous map $f : A \to S^2 - a - b$ such that f is nullhomotopic, not injective and a, b lie in the different component of $S^2 - f(A)$, where A is a compact set.
(3) Let X be a topological space with the property that, every continuous map $f : X \to X$ has a fixed point. Show that each retract Y of X has the property too.
(4) Let A be a compact contractible subspace of S^2. Show that A does not separate S^2.
(5) Show that if $g : S^2 \to S^2$ is continuous and $g(x) \neq g(-x)$ for all x, then g is surjective.
(6) Let G be a finitely generated abelian group. Give an example of topological space whose fundamental group is G.

Problem 3. Solve any three questions. (5+5+5)

(1) Let G be a topological group. Show that $\pi_1(G, e)$ is a commutative (abelian) group
(2) Compute the fundamental group of sphere with north and south pole identified.
(3) Compute the fundamental group of $\mathbb{R}^3 - C$, where C is a circle.
(4) Let $\mathbb{P}^2$ denotes the projective plane. Show that $\mathbb{P}^2$ is path connectet. Also compute $\pi_1(\mathbb{P}^2, x)$ where $x \in \mathbb{P}^2$.