COMMUTATIVE ALGEBRA

MTH 422/523, SEMESTER II, 2021-2022

Course Information

- **Instructor:** Dr. Sanjay Kumar Singh <sanjayks@iiserb.ac.in>
- **Office:** 210, Academic Building 1.
- **Email:** sanjayks@iiserb.ac.in.
- **Webpage:** http://home.iiserb.ac.in/∼sanjayks.
- **Class Time:** Monday, Wednesday $3.00 - 3.50$ PM and Thursday $4.00 - 4.50$ PM (can be changed in the case of online classes).

The aim of this course is to introduce commutative algebra. This theory has developed not just as a standalone area of algebra, but also as a tool to study other important branches of Mathematics including Algebraic Geometry and Algebraic Number Theory.

Analytic number theory mainly deals with enumeration and estimation of number-theoretic objects using tools from real and complex analysis. An important example of such an estimation is the Prime Number Theorem which states that the number of primes up to a certain x is asymptotic to $x/logx$. Likewise, an important enumerative result is the Dirichlet theorem on primes in arithmetic progressions, which states that for any positive coprime integers a and b , there are infinitely many primes of the form $a + nb$.

This course introduces the basic terminology, techniques and results in analytic number theory and culminates in a study of Prime Number Theorem and Dirichlet theorem on primes in arithmetic progressions.

Syllabus: The official Course Syllabus is as given in the Course Contents booklet

$$http://acad.iiserb.ac.in/cc/mth422.php$$

Course Contents:

- Arithmetic functions: the functions μ, ϕ, λ, σ and Λ; big O and little o notation, Euler's partial summation formula, some asymptotic formulas.
- Elementary results on distribution of prime numbers: Chebyshev's inequality, Bertrand's postulate, Mertens estimates, Selberg's asymptotic formula.
- Dirichlet's theorem on primes in arithmetic progressions: Dirichlet characters, infinitude of primes of the form $4n - 1$ and $4n + 1$, proof of Dirichlet's theorem.
- Dirichlet series: Convergence and other analytic properties, Euler products, Landau's theorem, Multiplication of Dirichlet series, Perron's formula.
- Riemann Zeta function and Dirichlet L-functions: gamma function, analytic continuation and functional equation.
- Analytic proof of the Prime Number Theorem.

Textbook:

- Introduction to analytic number Theory by T. M. Apostol, Springer.

Reference books:

- A course in analytic number theory, by M. Overholt, AMS.
- Problems in analytic number theory, by M. Ram Murty, Springer
- Methods in number theory, by M. Nathanson, Springer.
- Anaytic number theory, by H. Iwaniec and E. Kowalski, AMS.

Assignment. There will be 4 assignment in this course. The homework assigments will be posted on the course webpage. Assignment problems will give you an idea about the examination. You are encouraged to solve all problems given in the text book.

Home work and class exercise. In every class you will get some home work which you don't need to submit. You can discuss it to me.

Grading Policy: The grading policy for the 2021-22-II Semester is divided into 4 following components

- Participation of students in the course (10%)
  (It will be based on your attendence/participation in the class and few surprise oral questions in the class)
- Quiz (30%) (2 Quizess each of 15 Marks)
- Mid Semester Examination (30%)
- Final Examination (30%)

Quiz: There will be two quizess in the semester. The dates will be announced in the class.

Office Hours: By appt.

***.** In case of any further questions regarding the course, please send me an email.